Post-Quantum Security of Lattice-based Cryptosystems

Rajendra Kumar

TQC 2025 Bengaluru, India

Lattices

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n : rank of the lattice

Lattices

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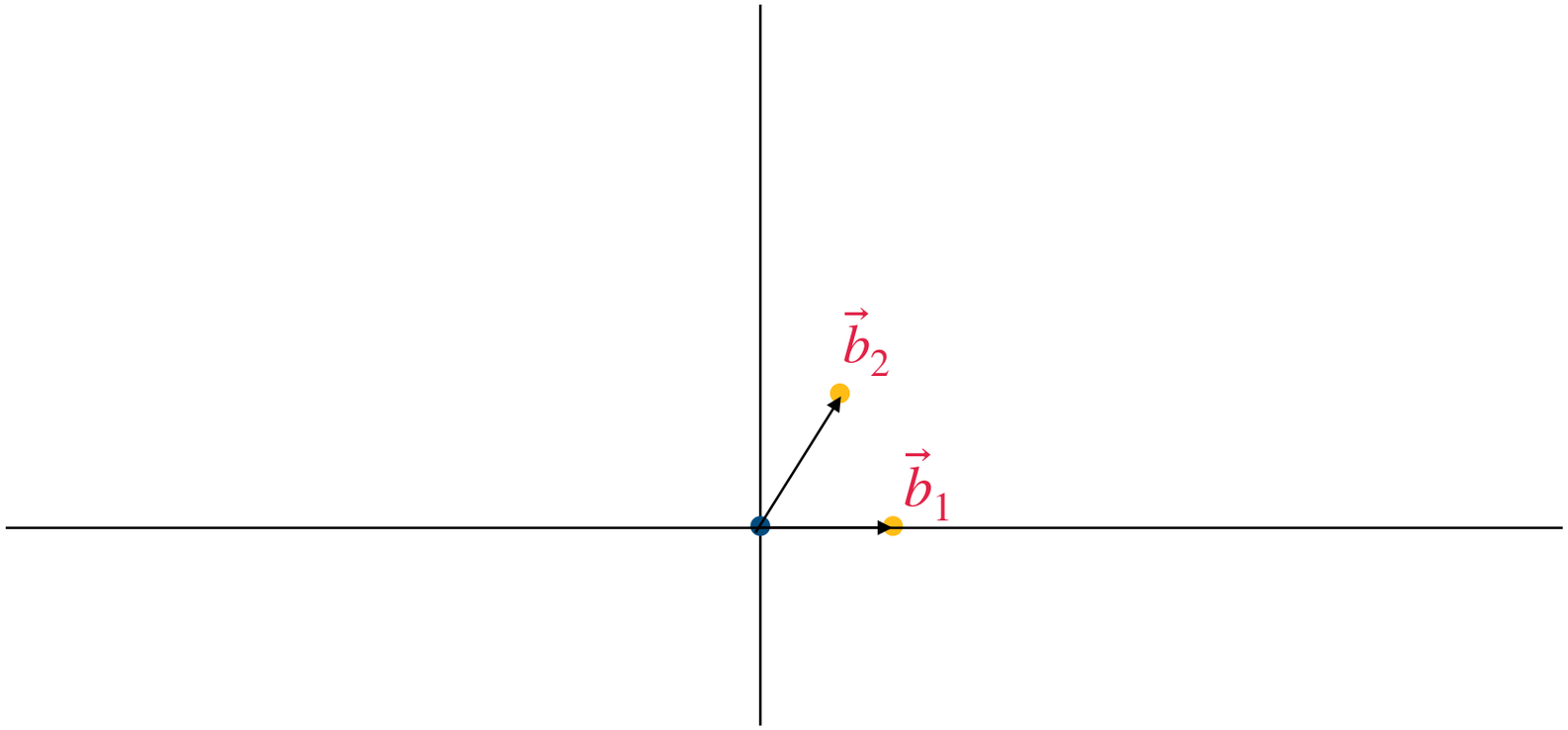

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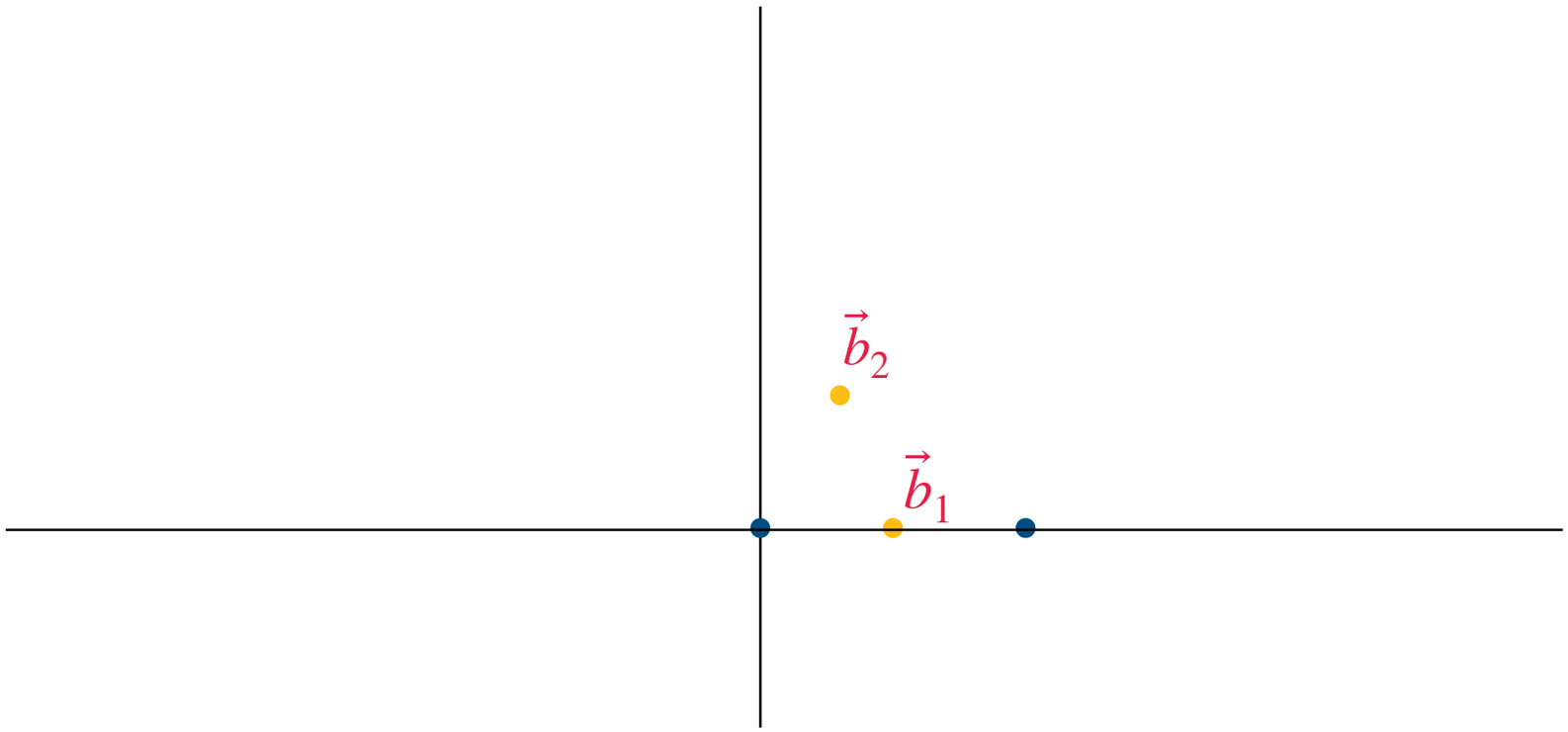

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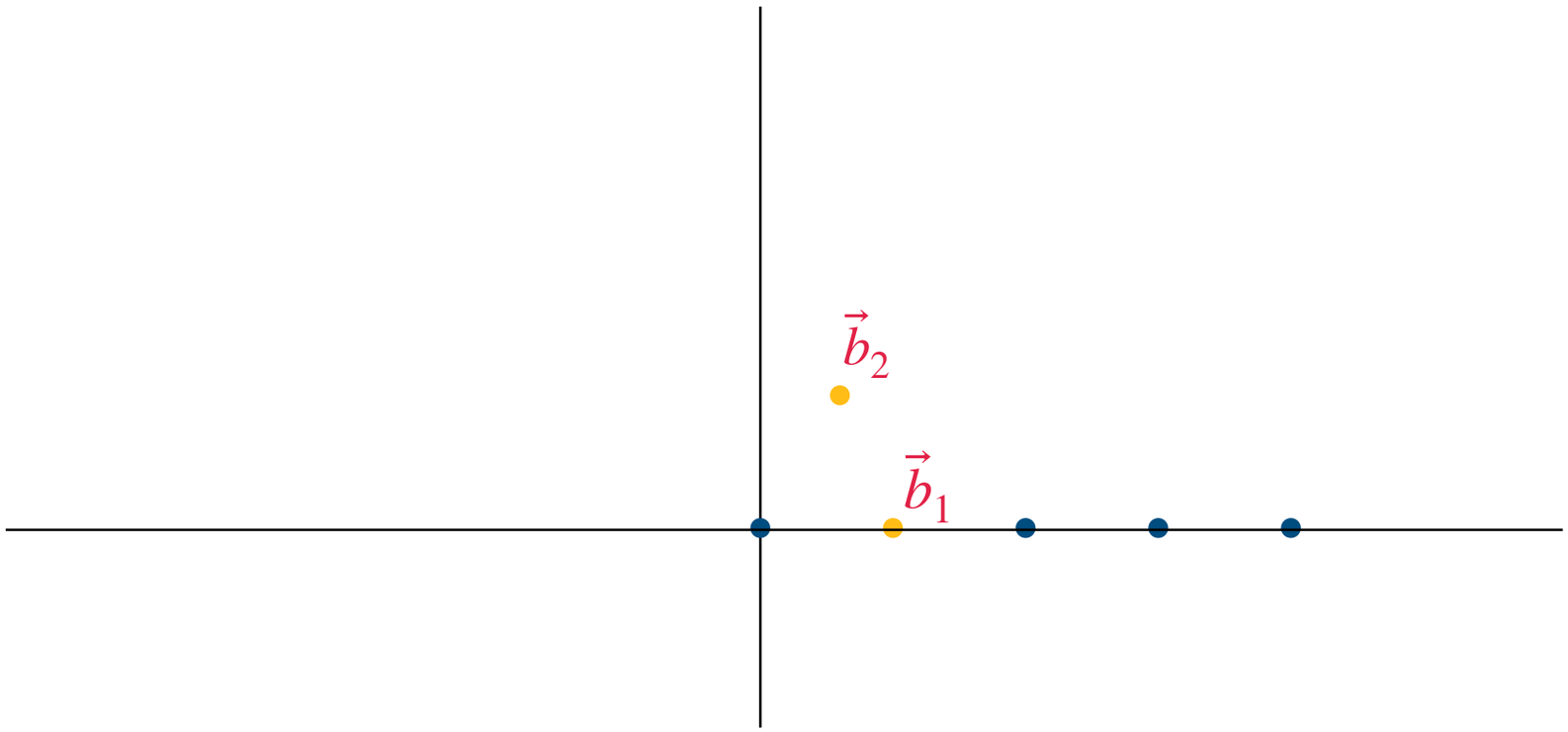

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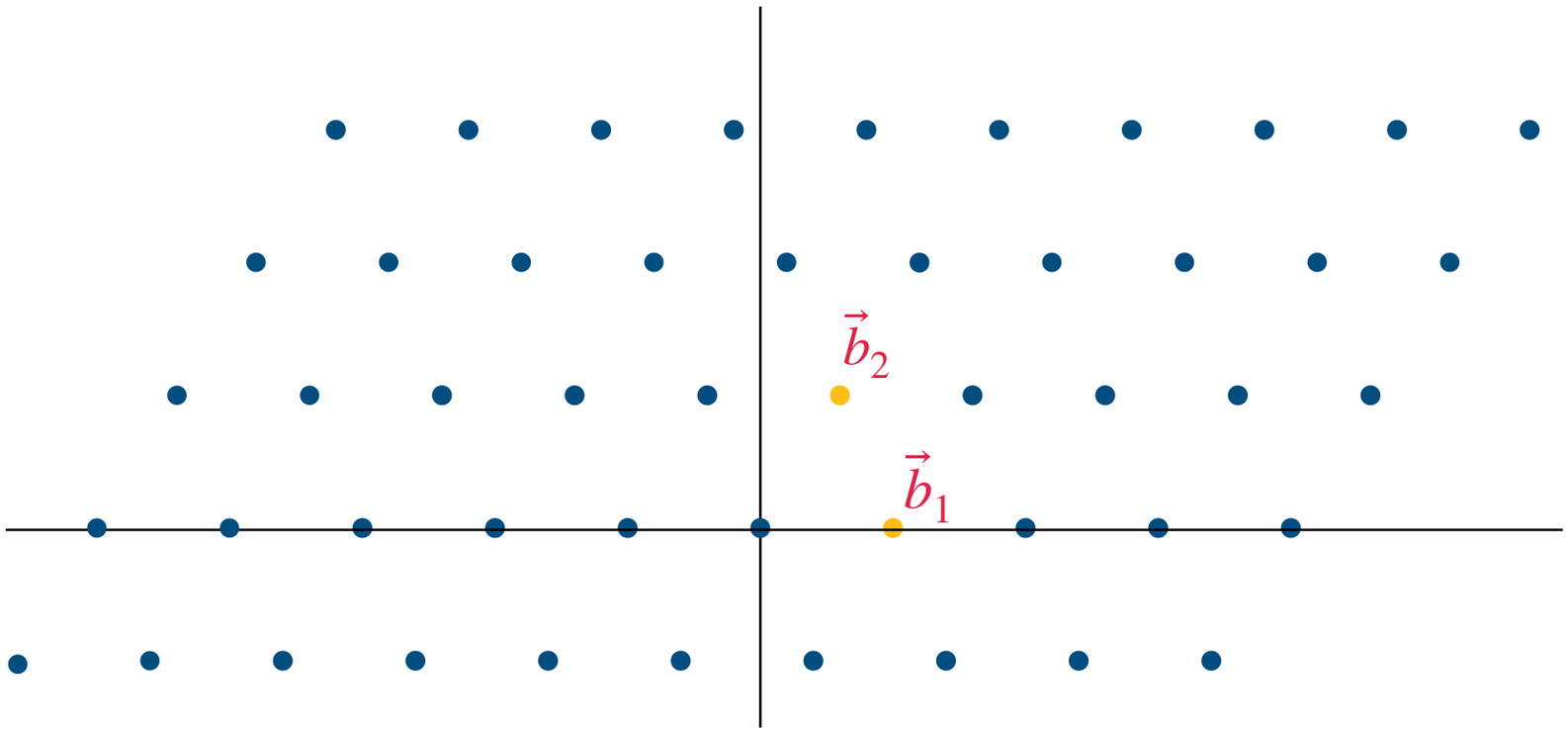

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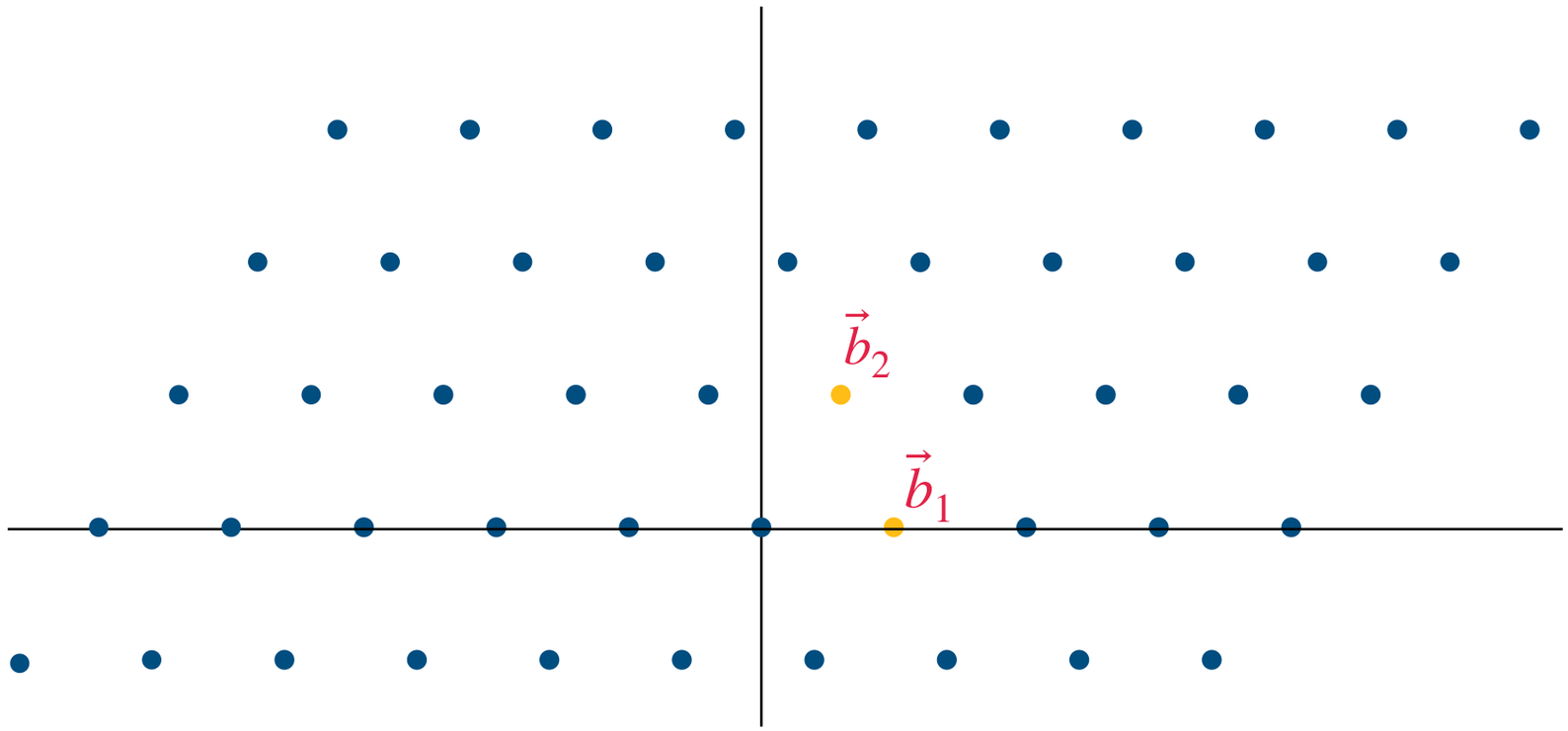

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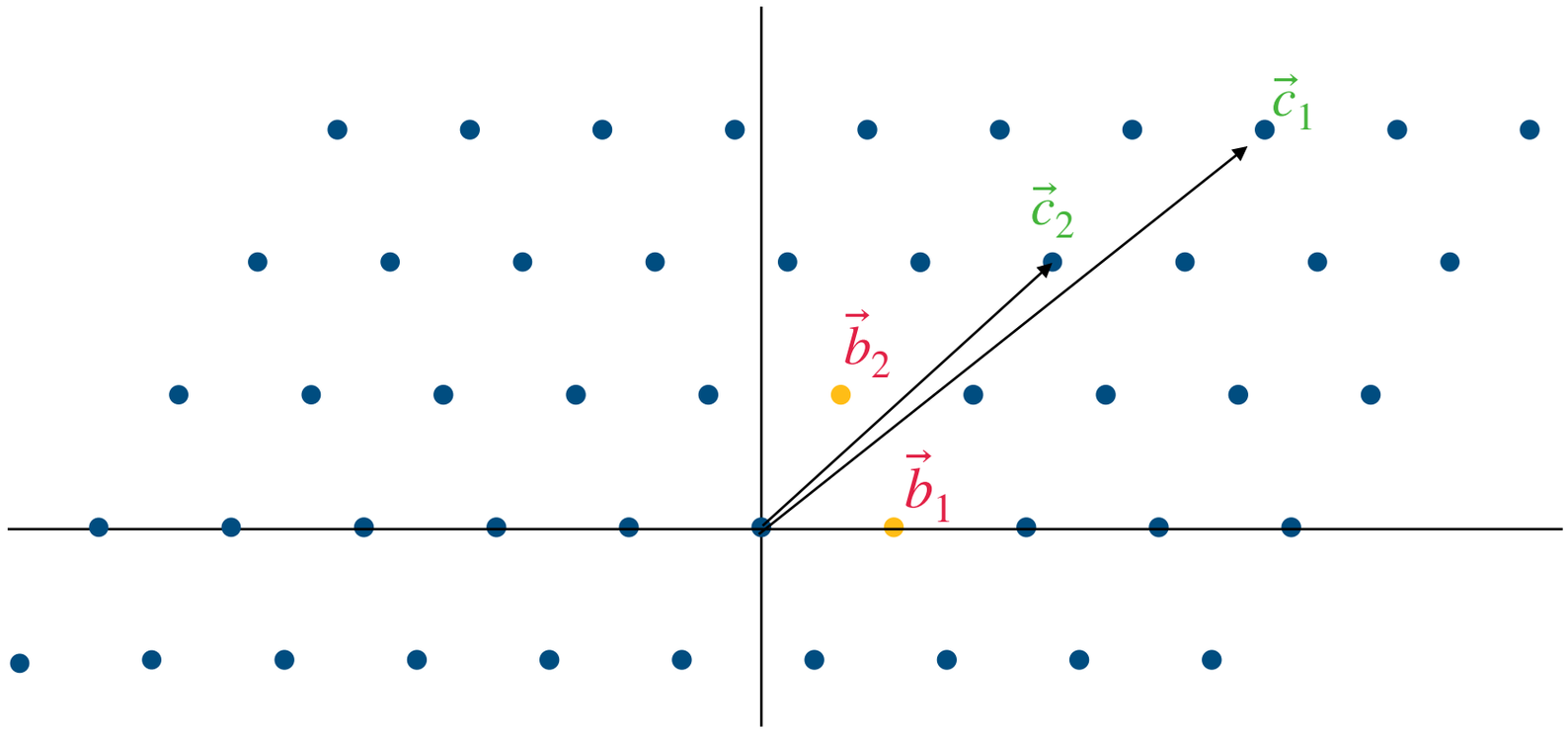
Rank n

Lattices



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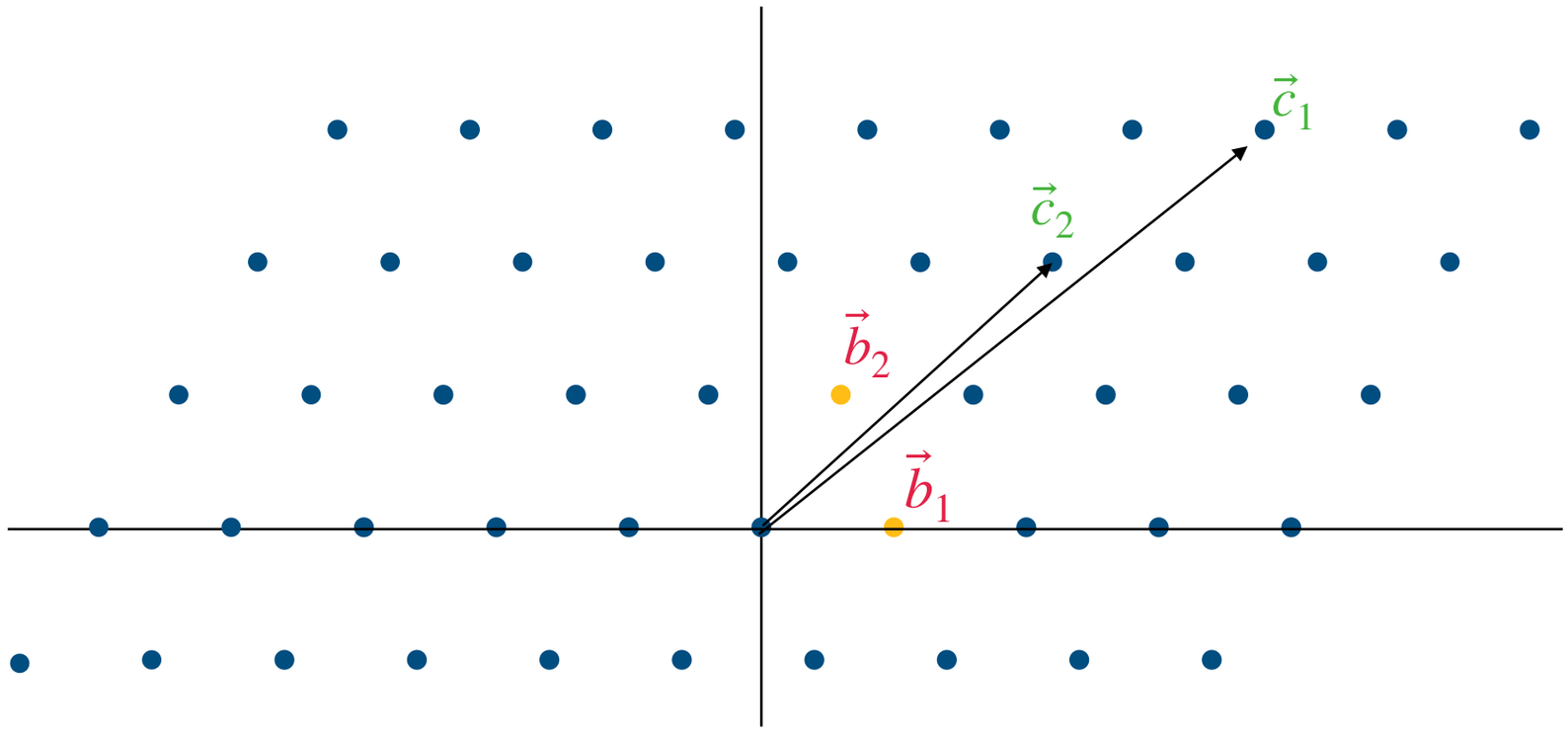

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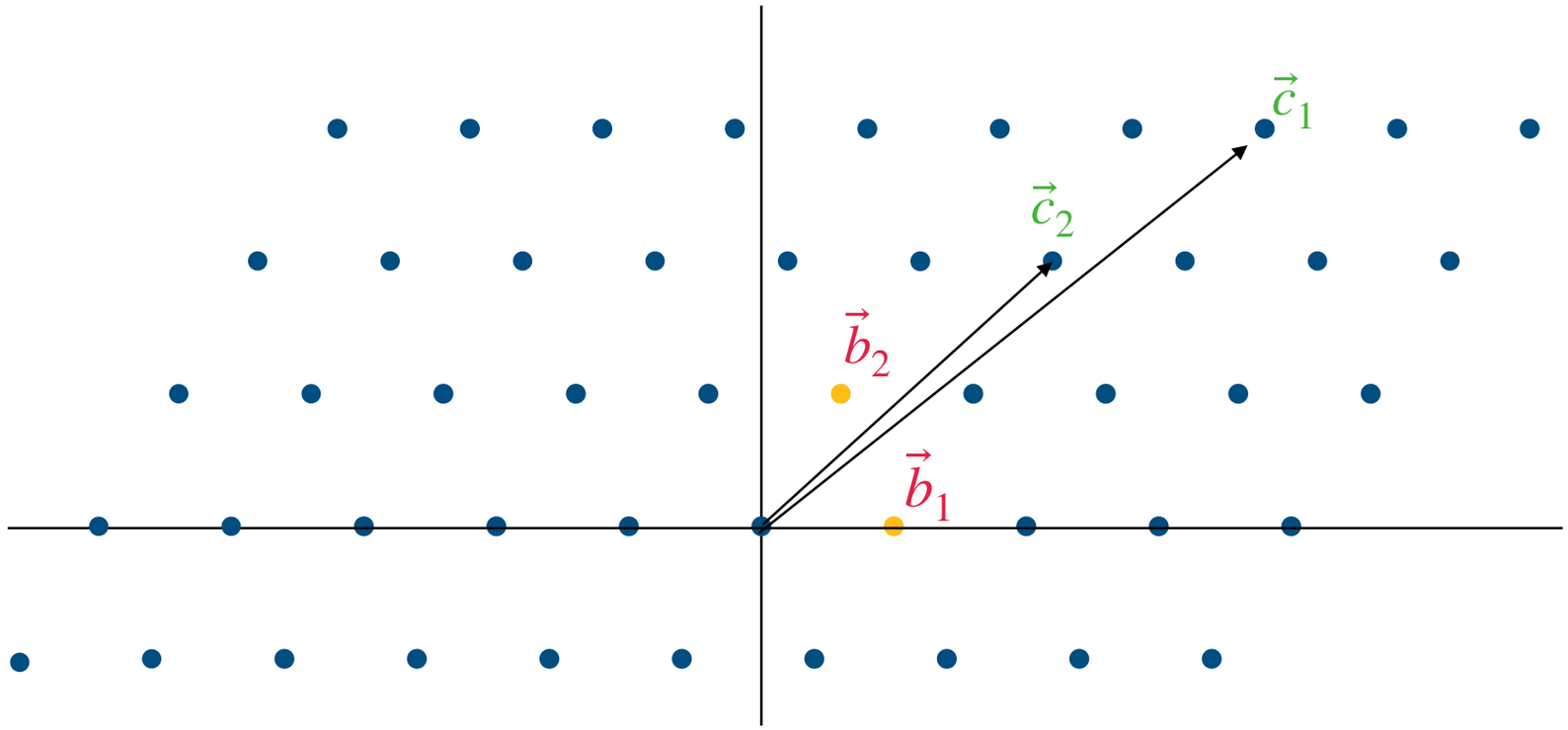

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γ-Shortest Vector Problem (SVP): $(\gamma \geq 1)$

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Goal: distinguish between

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For small γ, decision problem is as hard as search problem.

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γ-CVP is at least as hard as γ-SVP.

Lattice-based Crypto

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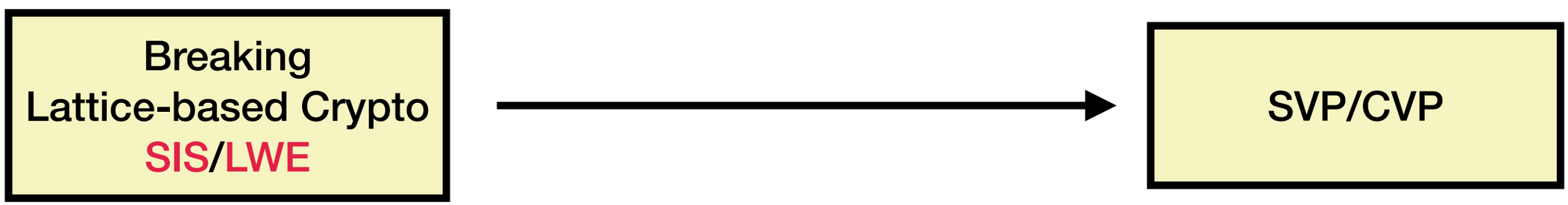
Breaking
Lattice-based Crypto
SIS/LWE

Lattice-based Crypto



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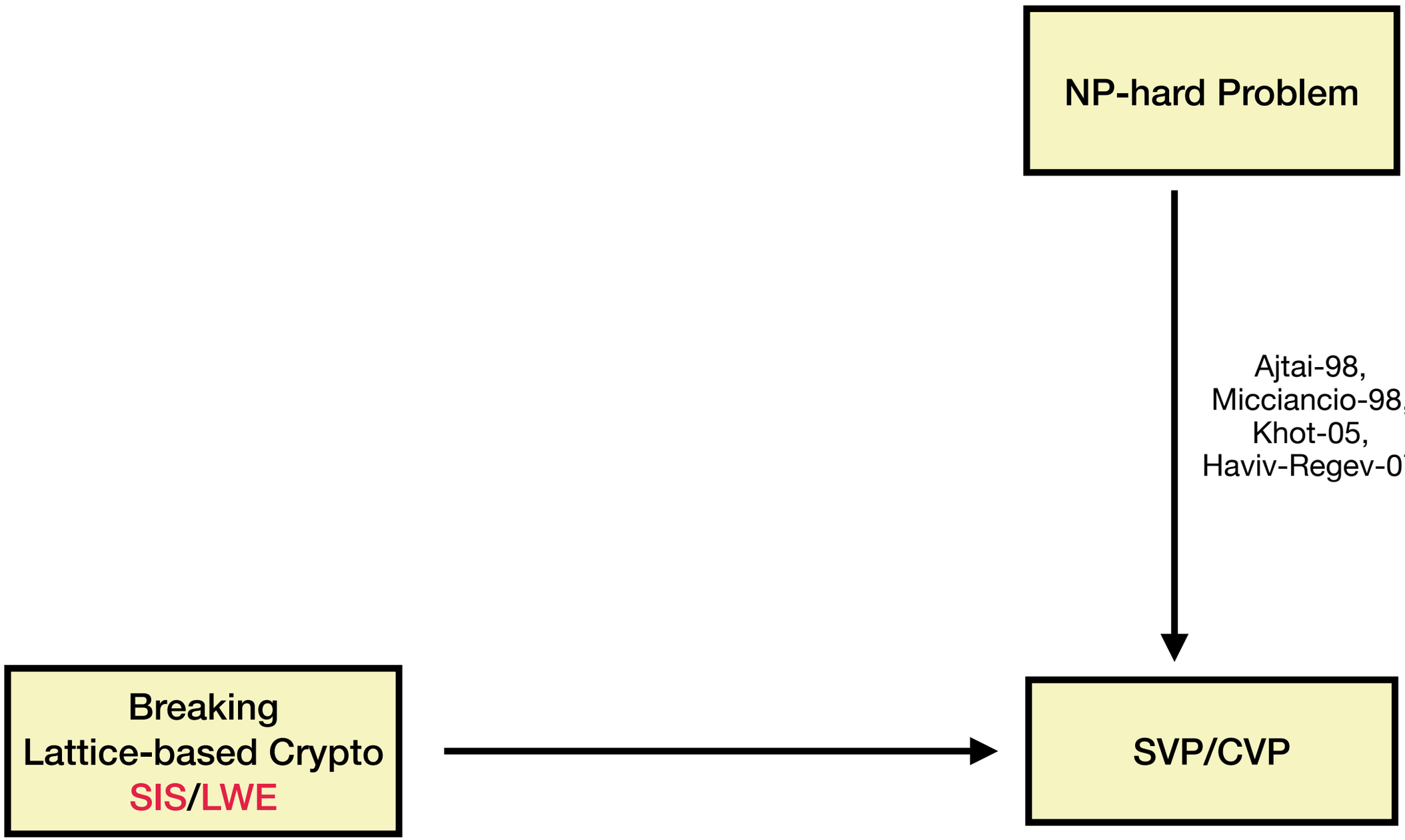

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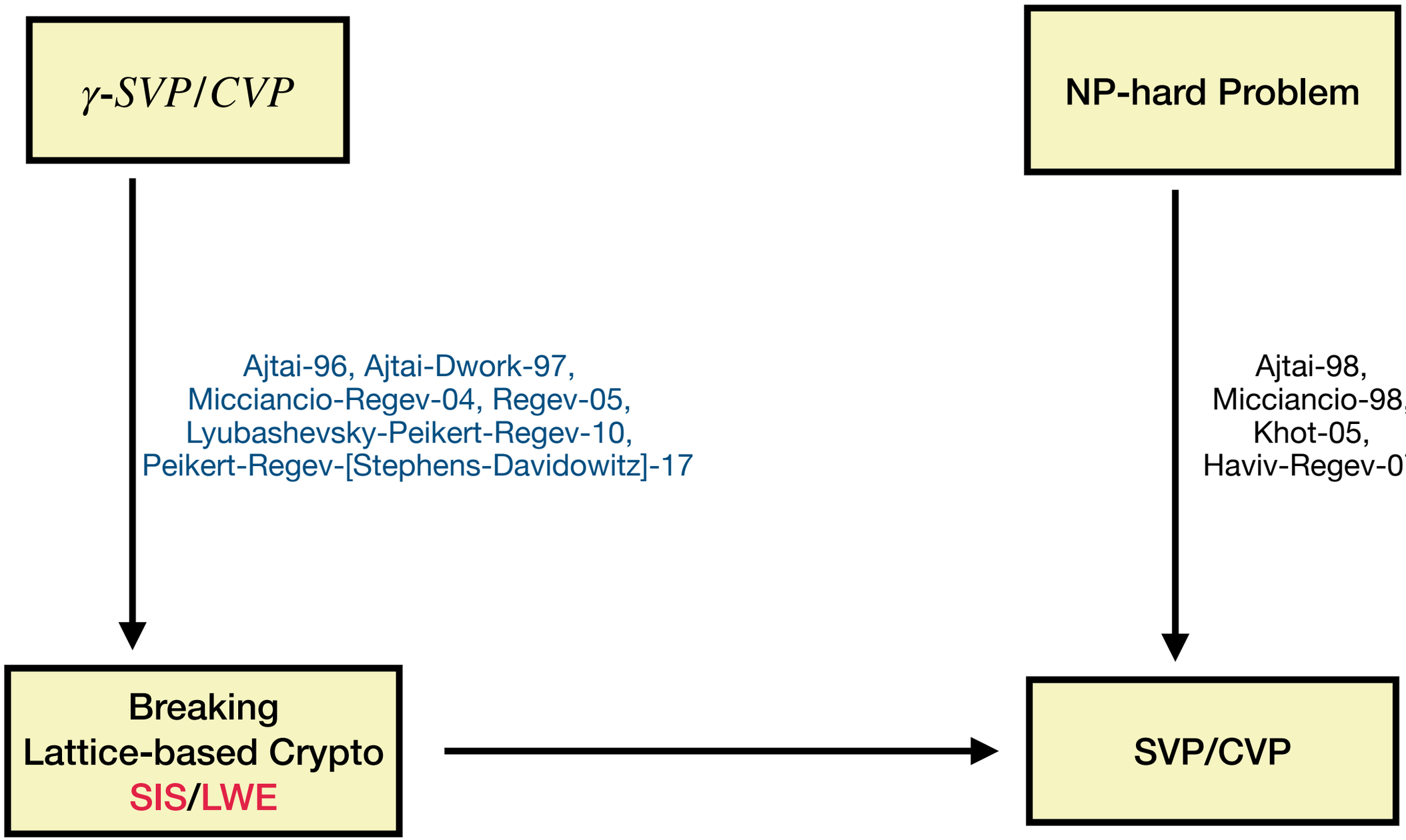

Lattice-based Crypto

γ-SVP/CVP

Is γ-SVP/CVP hard?

Ajtai-96, Ajtai-Dwork-97,
Micciancio-Regev-04, Regev-05,
Lyubashevsky-Peikert-Regev-10,
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NP-hard Problem

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Hardness of γ-SVP/CVP

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- γ-CVP is NP-hard for $\gamma < n^{1/\log\log n}$.

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- γ-SVP is poly-time hard under some reasonable conjecture for $\gamma < n^{1/\log\log n}$.

  [Dinur-03, Khot-05, Haviv-Regev-07, Micciancio-12, Bennett-Peikert-23]

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Crypto

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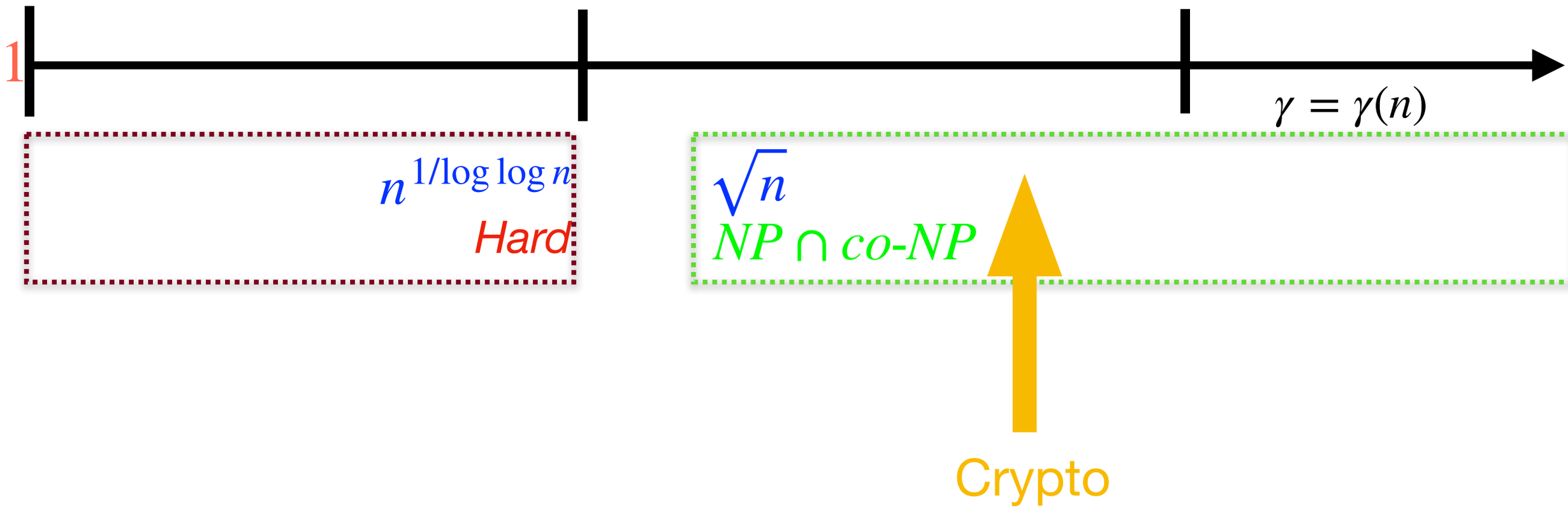

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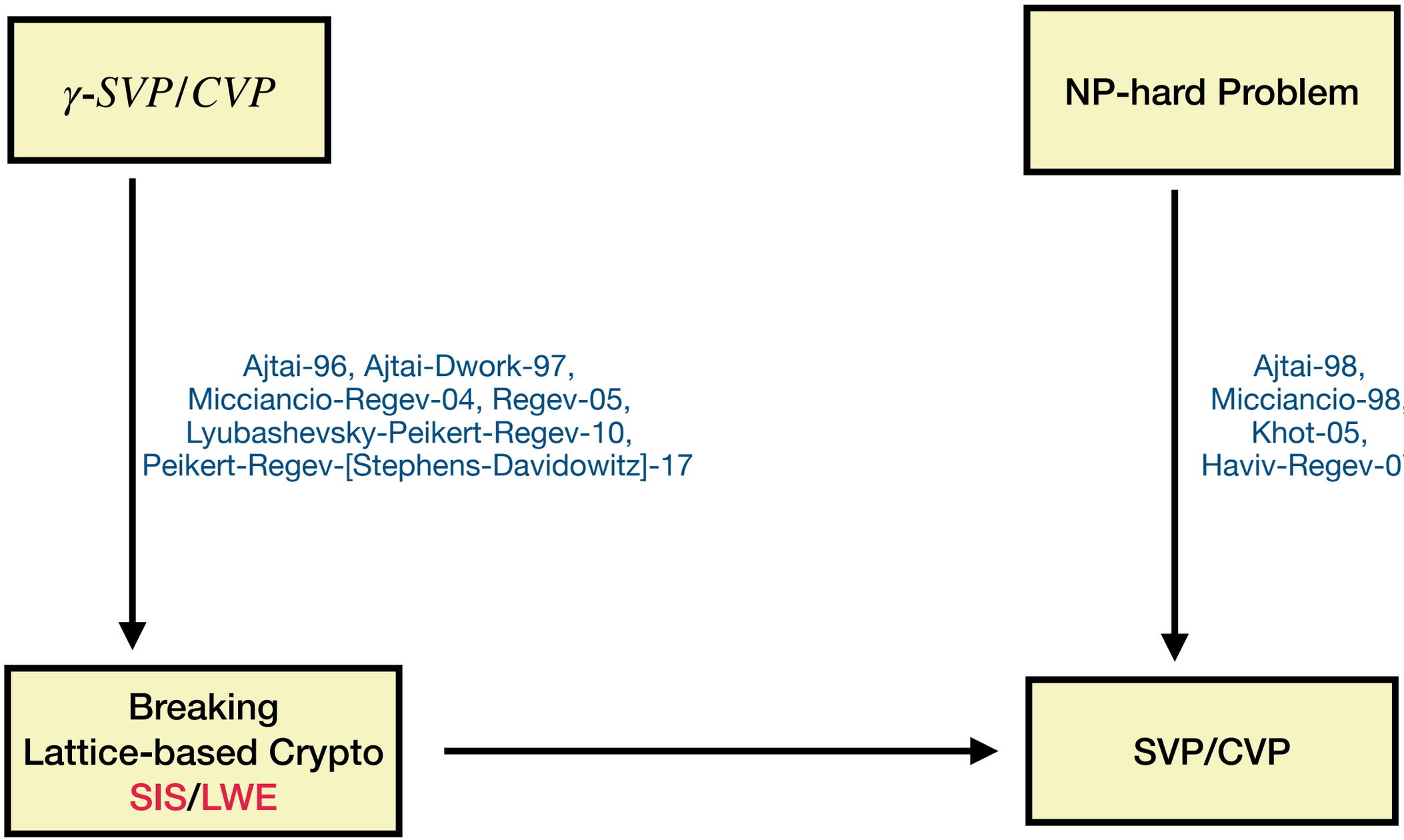

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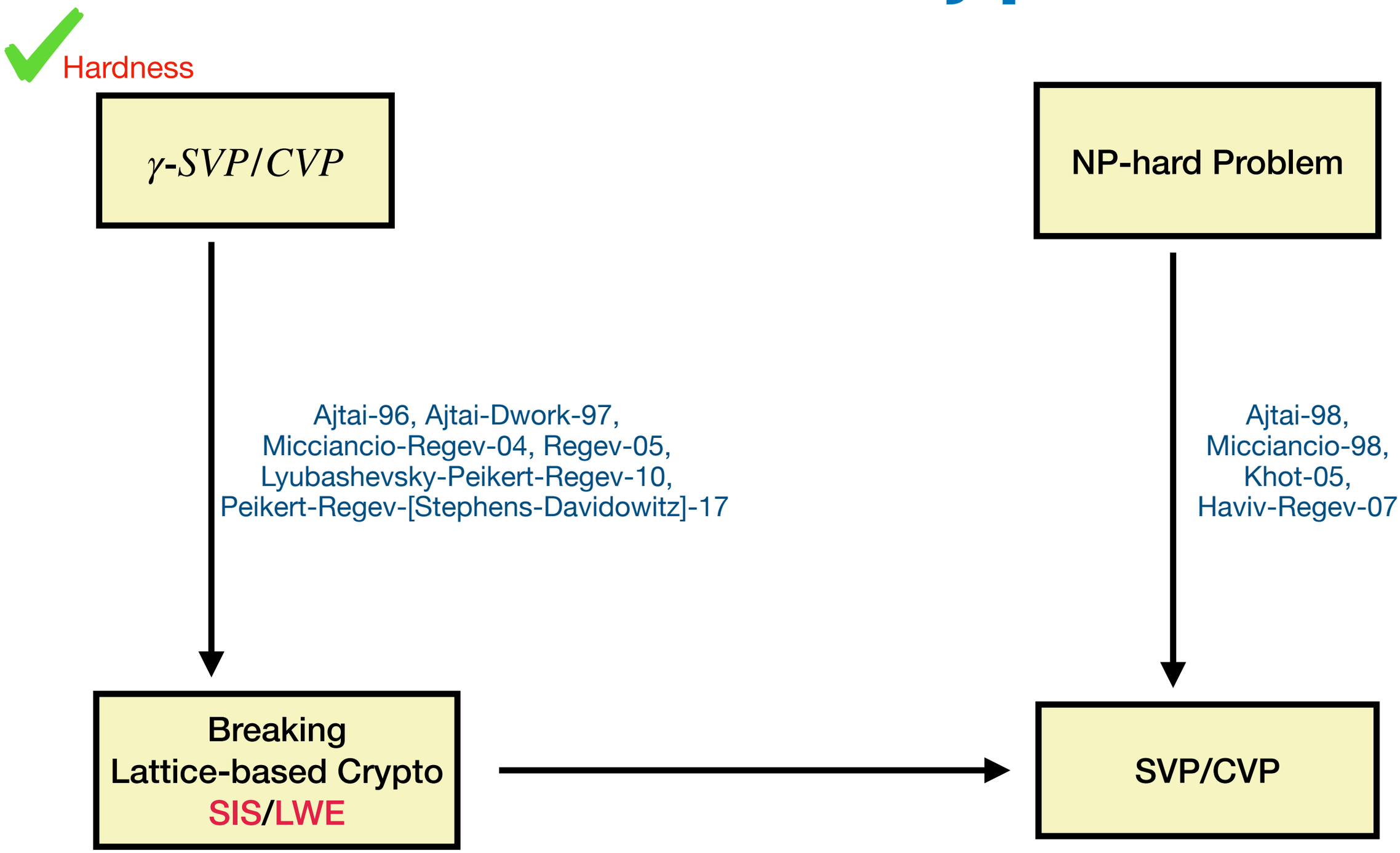

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Can we solve poly-approx SVP/CVP efficiently?

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Conjecture: $poly(n)$-SVP is $exp(\Omega(n))$-hard.

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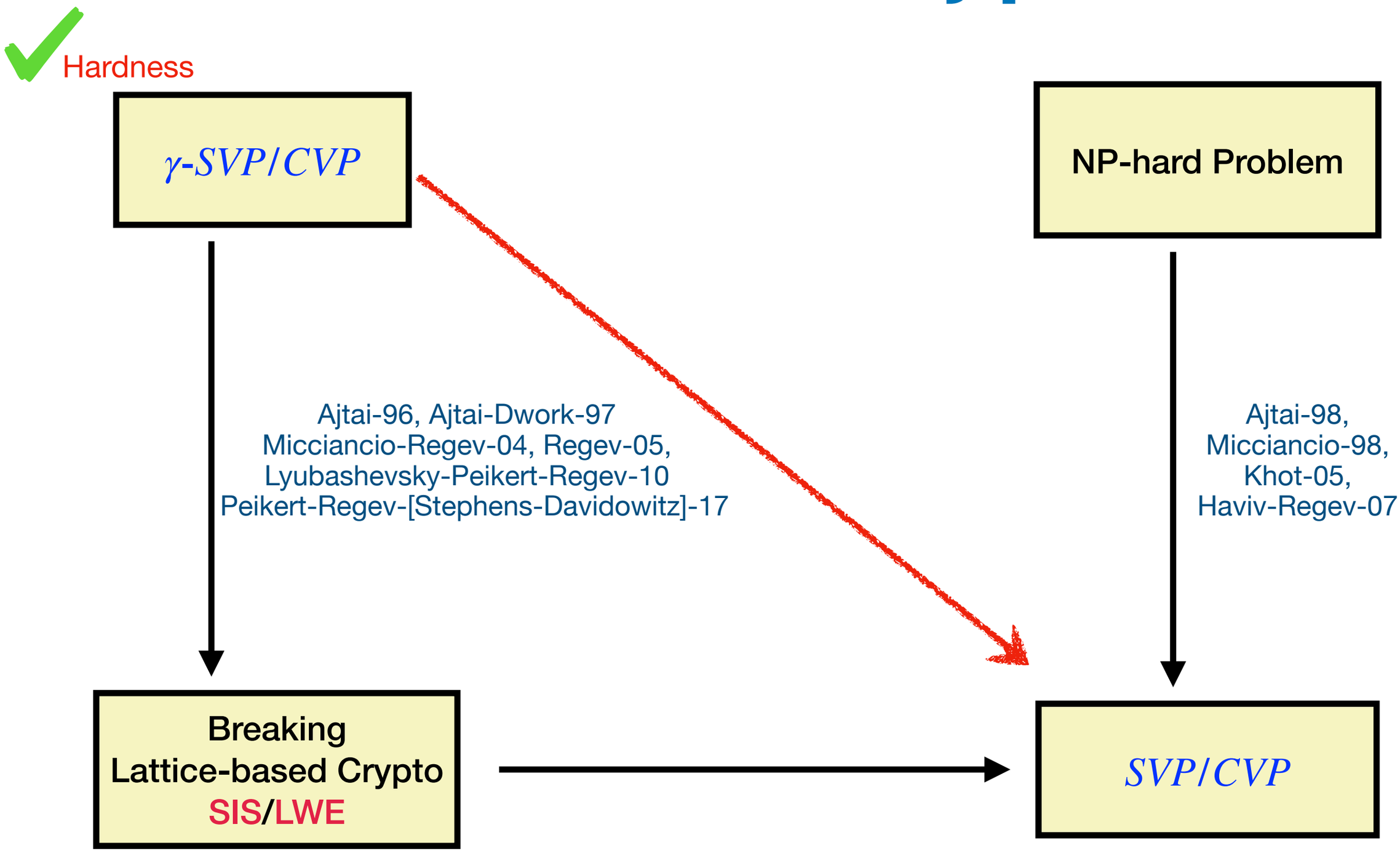

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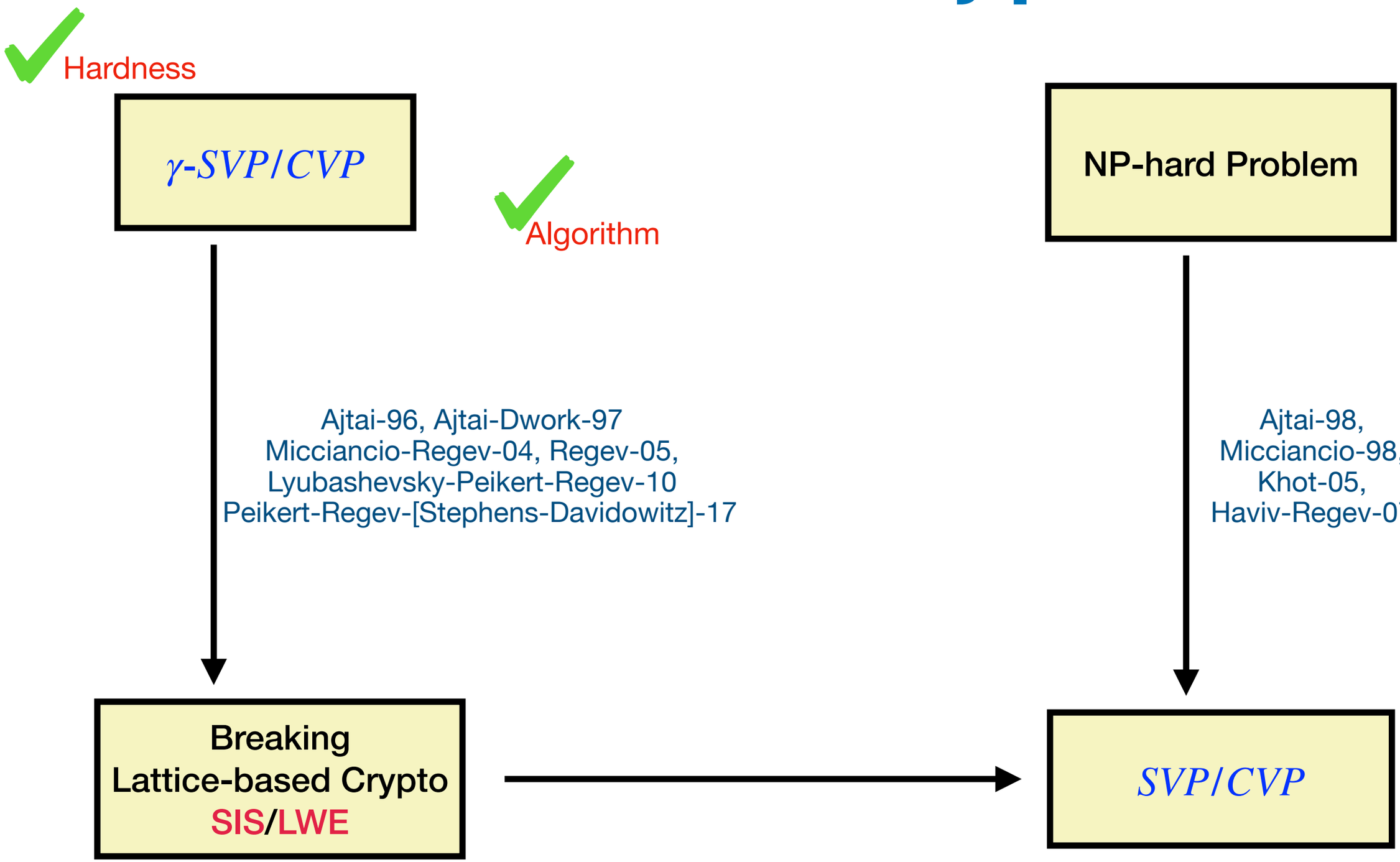

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$exp(n)$-hardness of γ-SVP/CVP?

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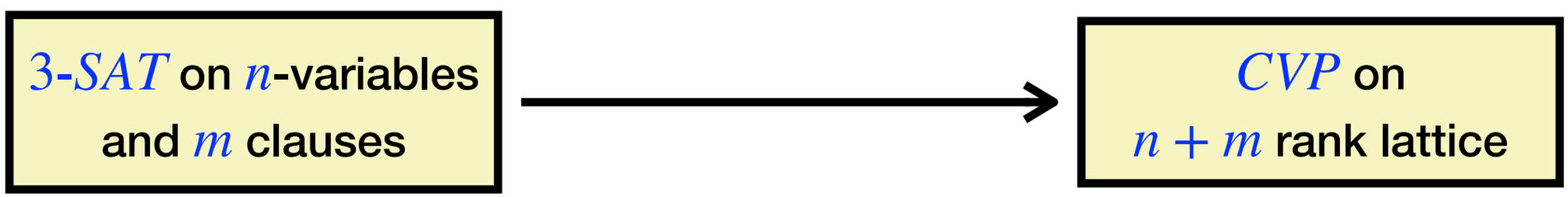

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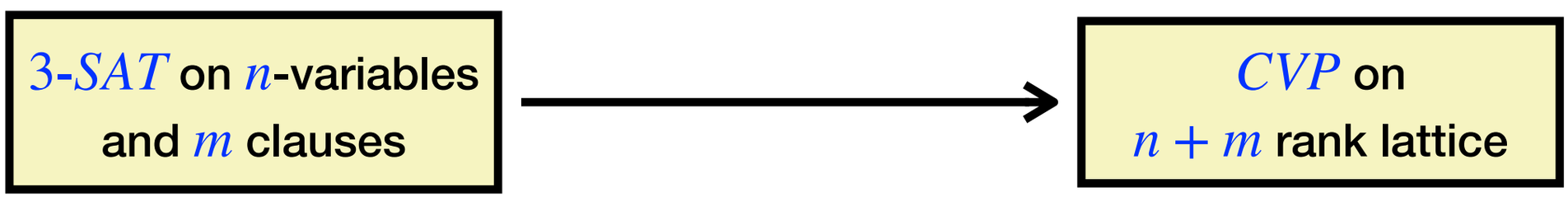

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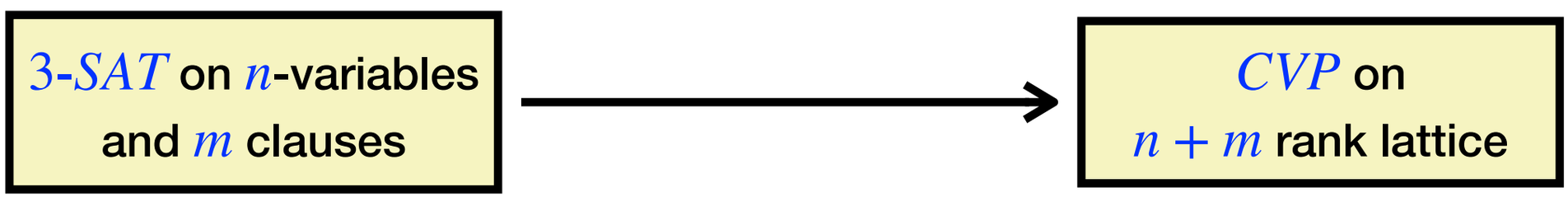

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Open Problem: $exp(\Omega(n))$-hardness of SVP.

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Can we get 2^{Cn}-hardness for CVP for some specific constant $C > 0$?

➡ It is impossible to get 2^{Cn}-hardness for CVP under **SETH/QSETH** via poly-time Turing reductions from k-SAT unless the polynomial hierarchy collapses to the third level.

[Aggarwal-Kumar 23]

Barrier for $exp(Cn)$-hardness

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➡ A compressed instance may not be of the same problem.

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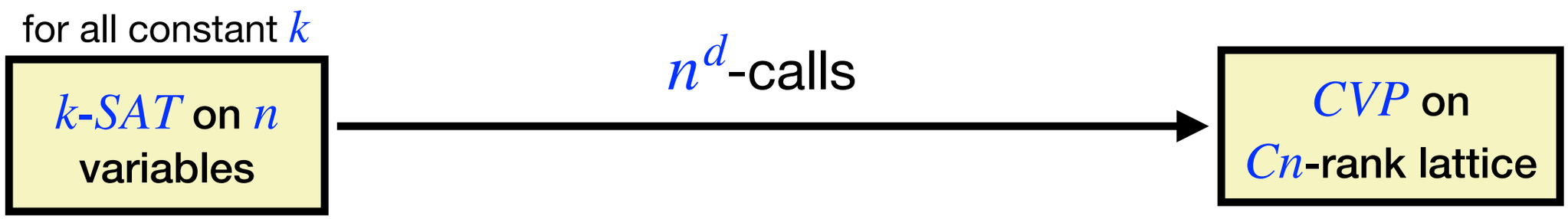

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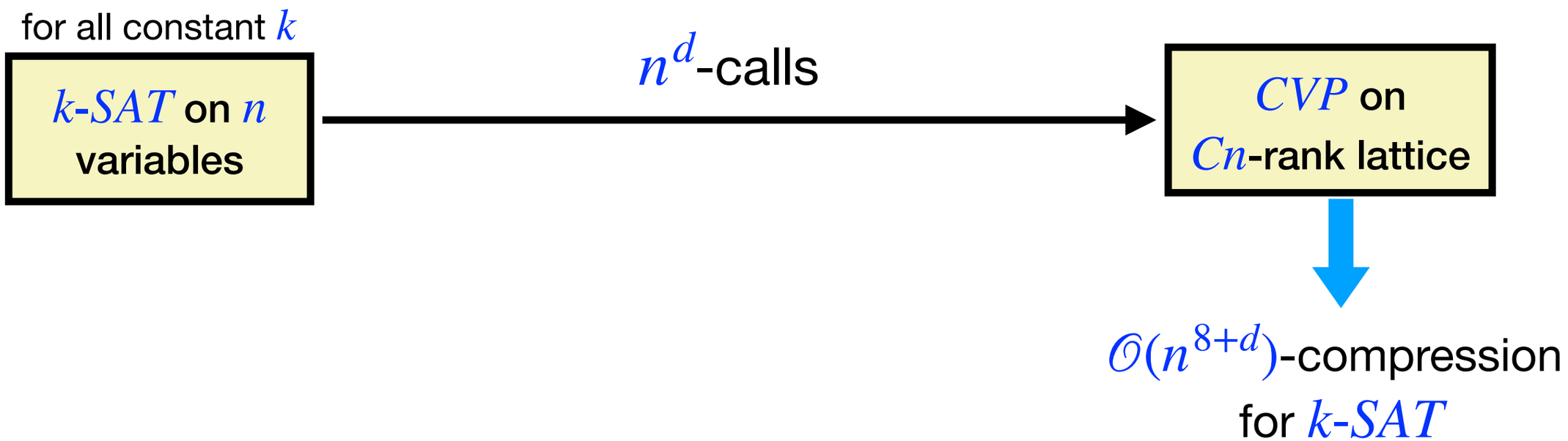

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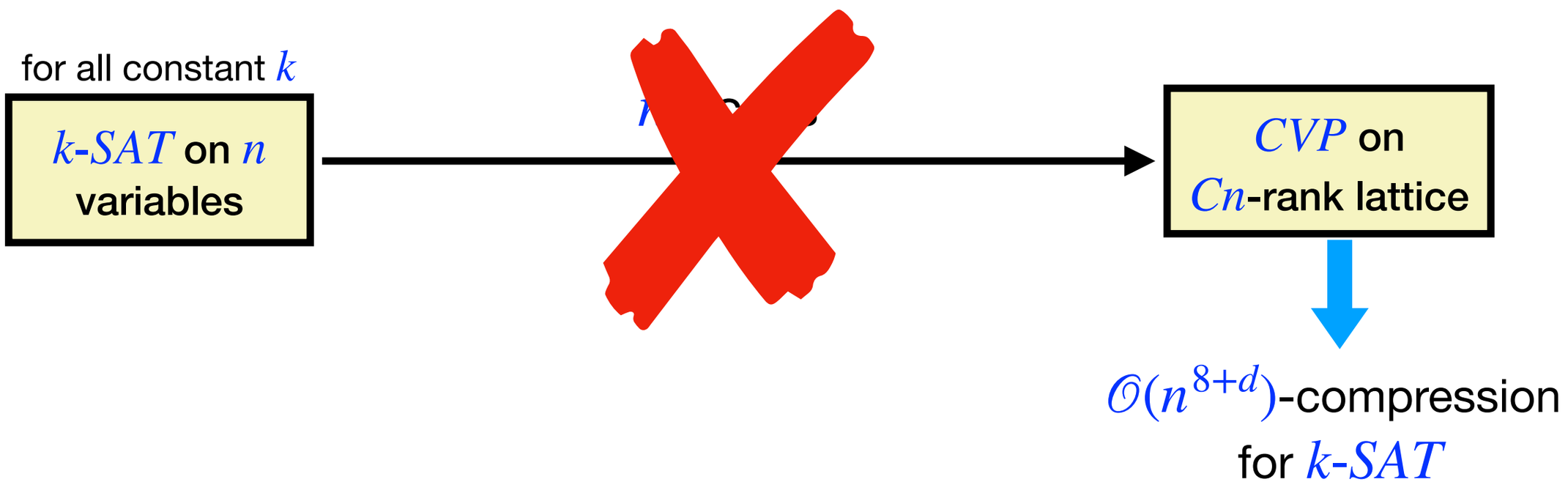

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CVP on n-rank lattice

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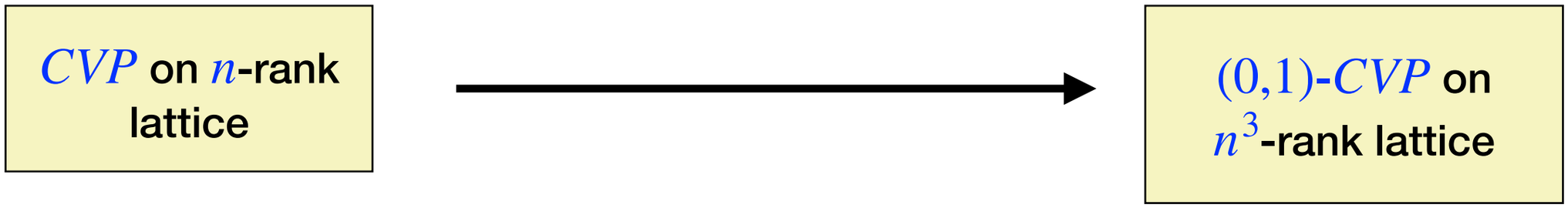

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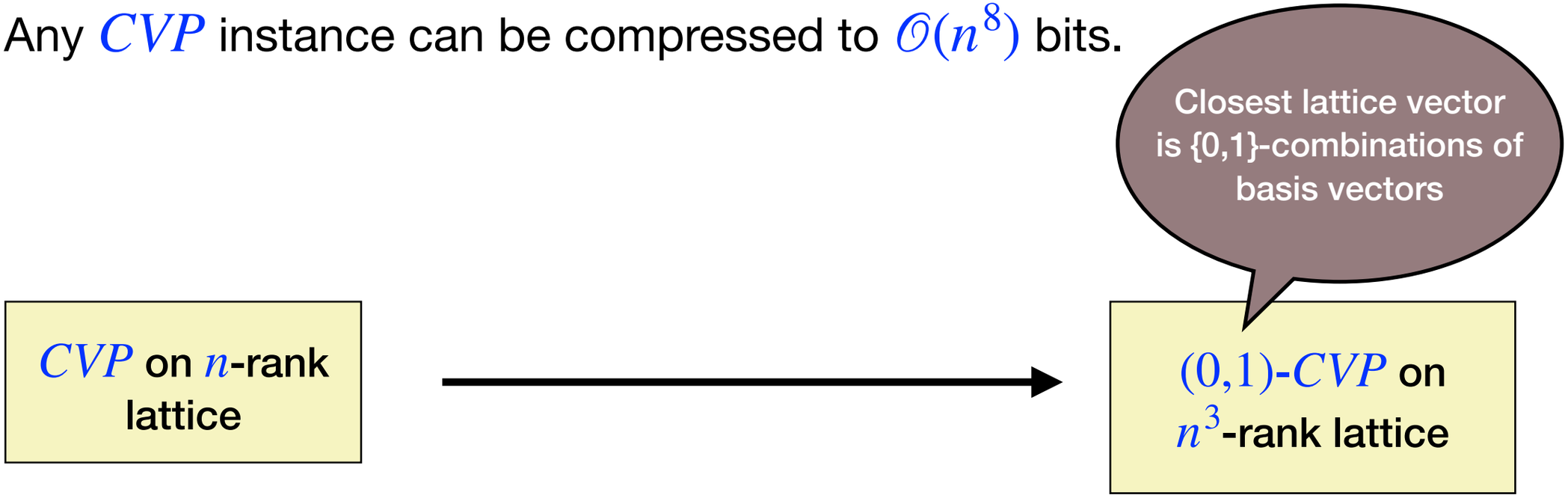

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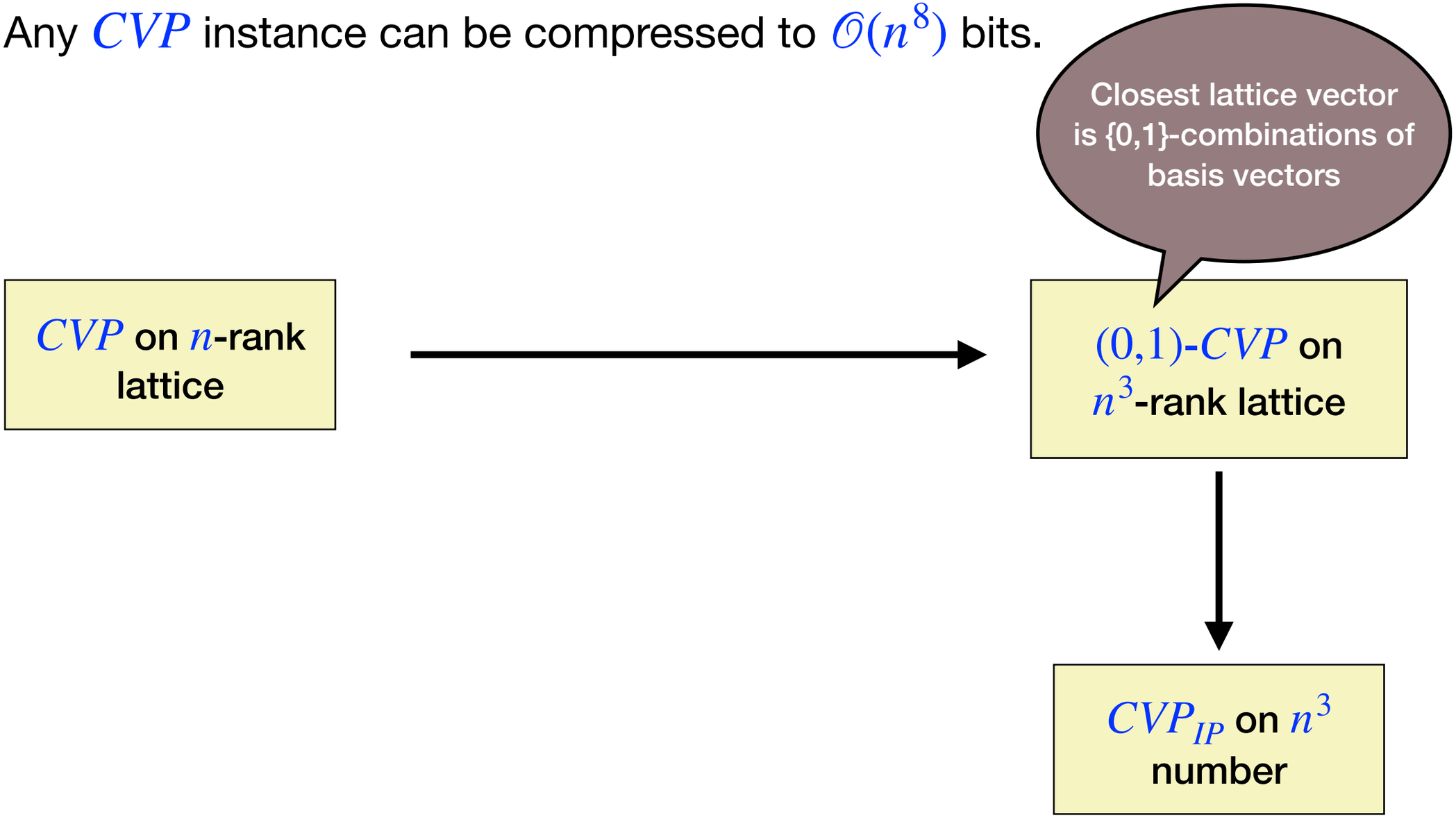

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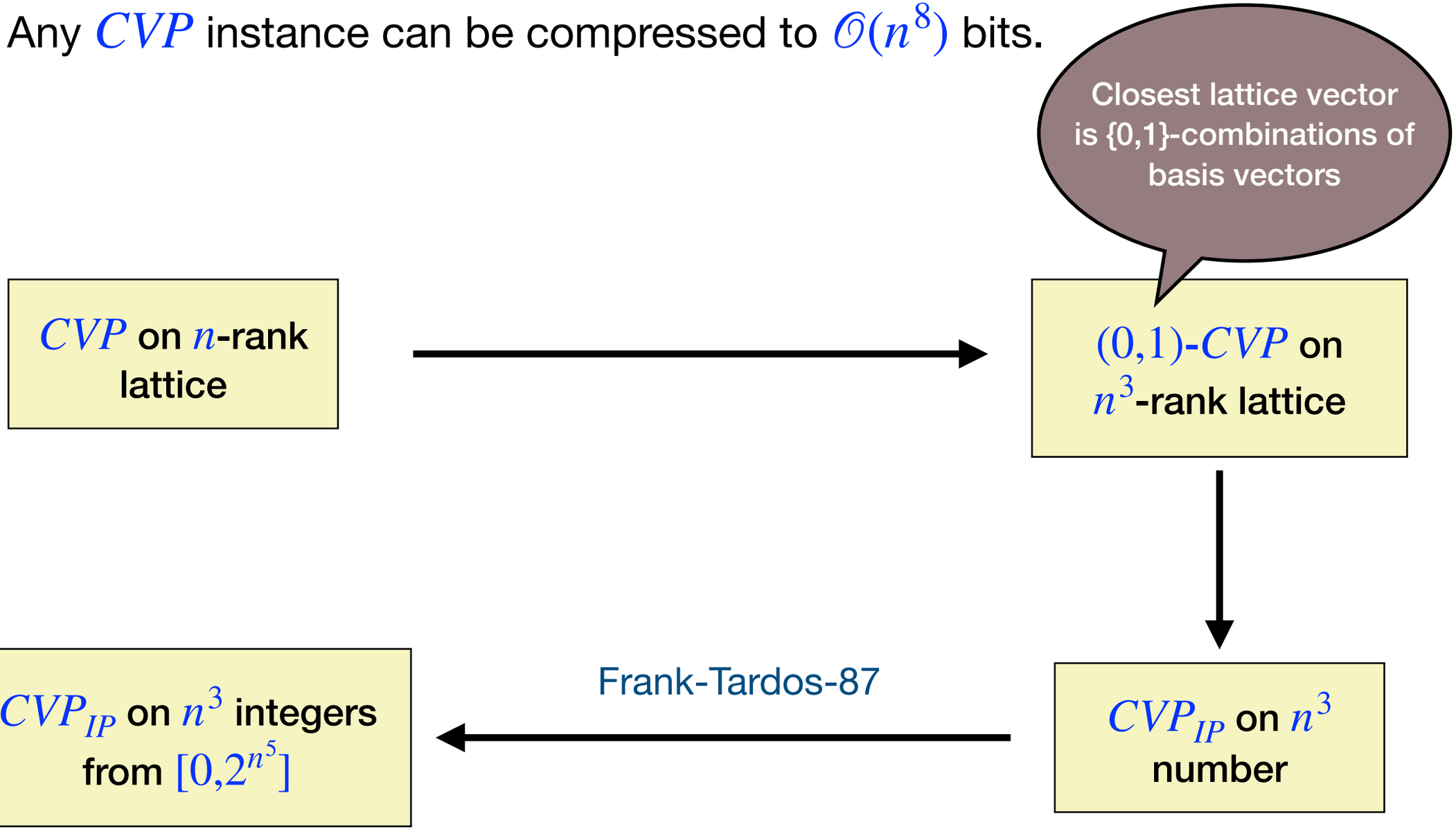


for all constant k
k-SAT on n variables
CVP on Cn-rank lattice

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Is CVP/SVP really that hard?

for all constant k

k-SAT on n variables

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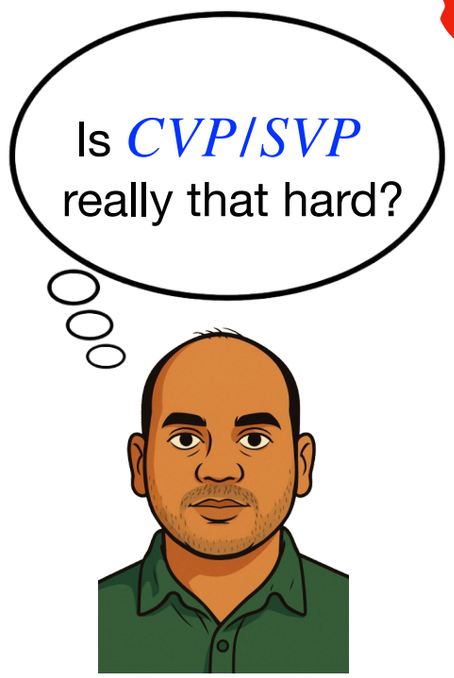
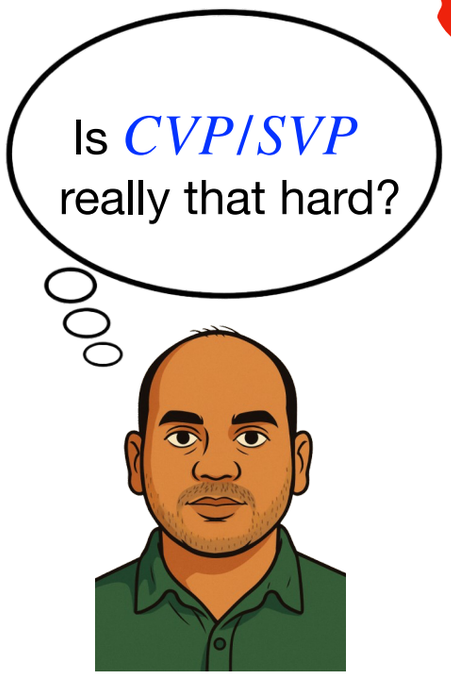


$Weighted$-Max-2-SAT on n-variables

for all constant k

k-SAT on n variables

CVP on Cn-rank lattice



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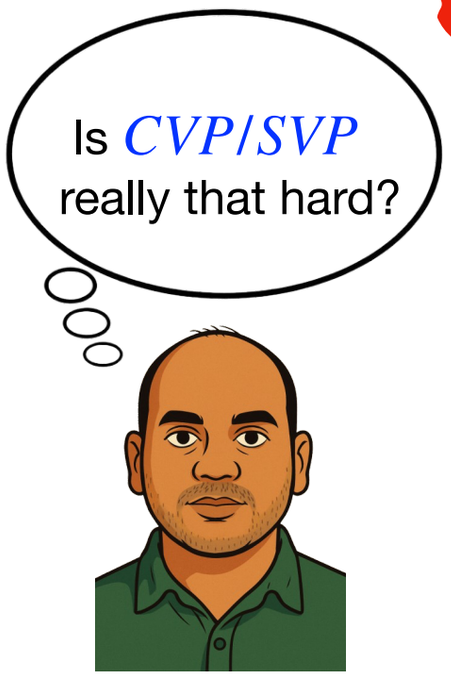

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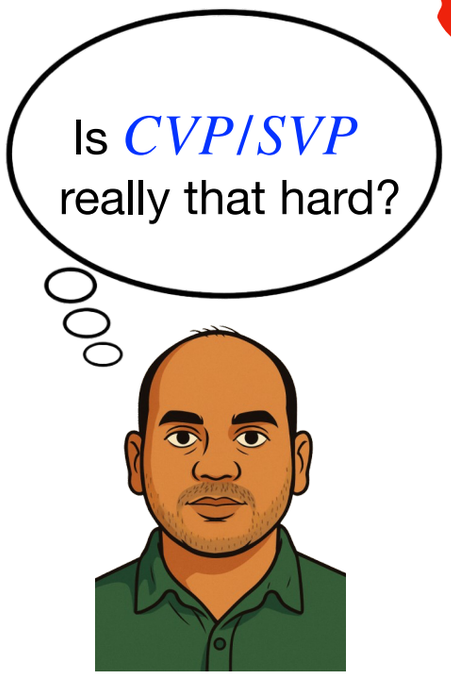

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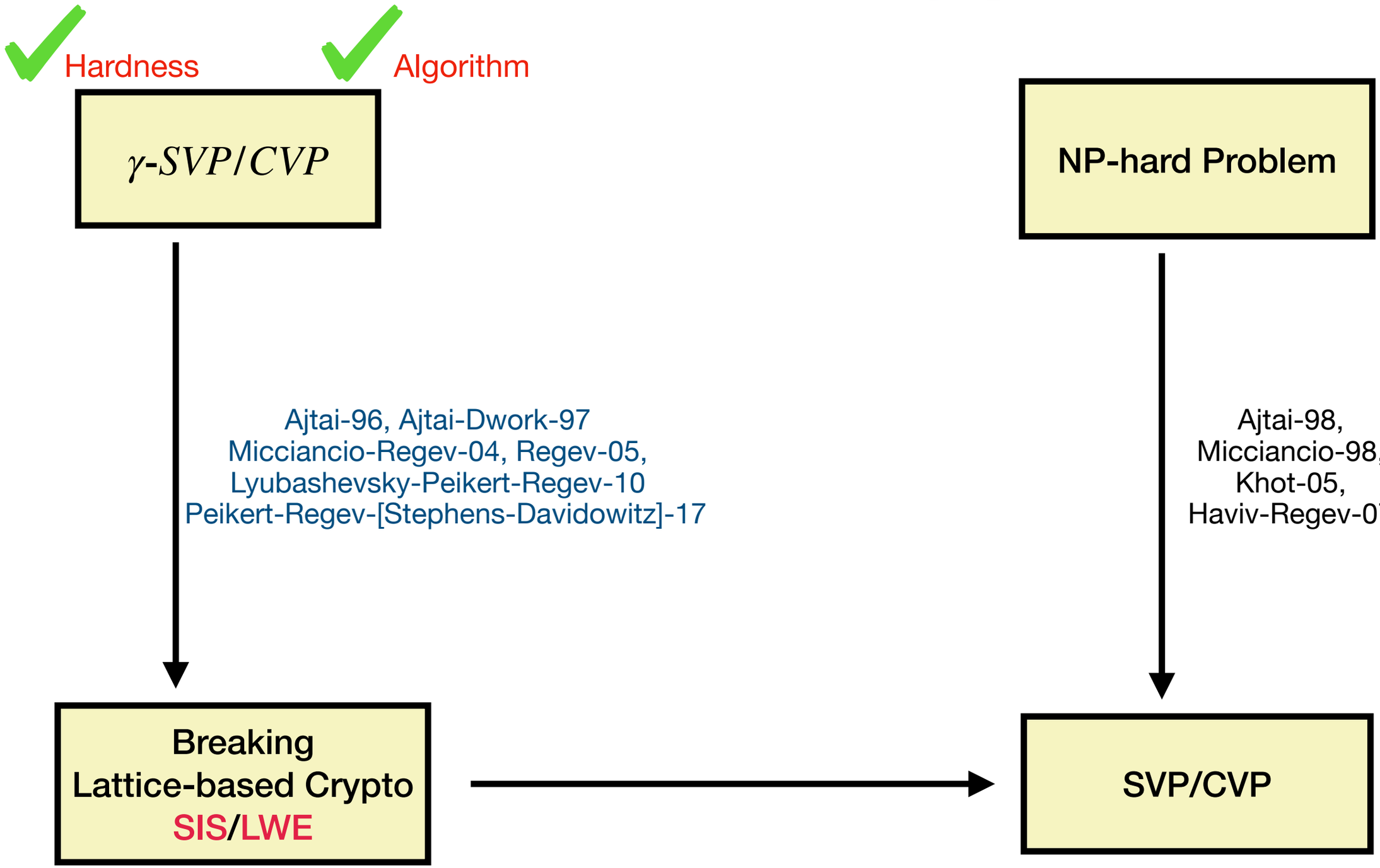
Open problem: Rank-preserving reduction from CVP to $(0,1)$-CVP

Lattice-based Crypto



Lattice-based Crypto

✓ Hardness ✓ Algorithm

γ-SVP/CVP

Can we build crypto on relatively hard problems?

Ajtai-96, Ajtai-Dwork-97
Micciancio-Regev-04, Regev-05,
Lyubashevsky-Peikert-Regev-10
Peikert-Regev-[Stephens-Davidowitz]-17

Breaking Lattice-based Crypto SIS/LWE

NP-hard Problem

Ajtai-98,
Micciancio-98,
Khot-05,
Haviv-Regev-07

SVP/CVP

LWE and SIS

➡️ Security of lattice-based crypto is equivalent to hardness of average-case lattice problems: Learning with Errors (LWE) and Short Integer Solutions (SIS).

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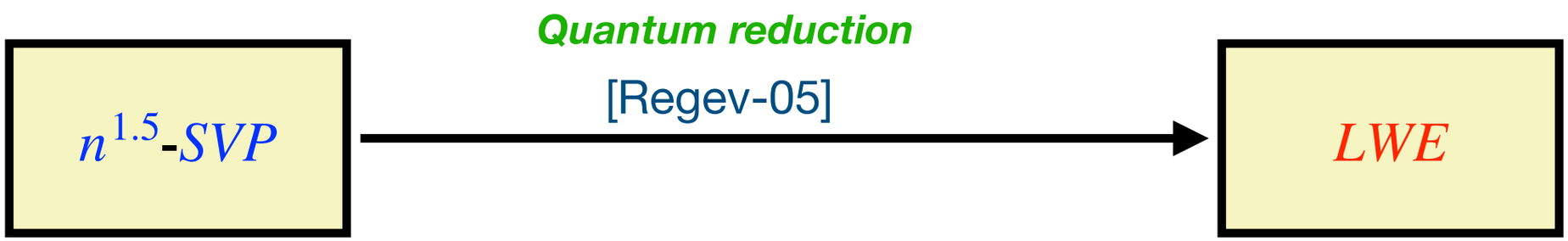

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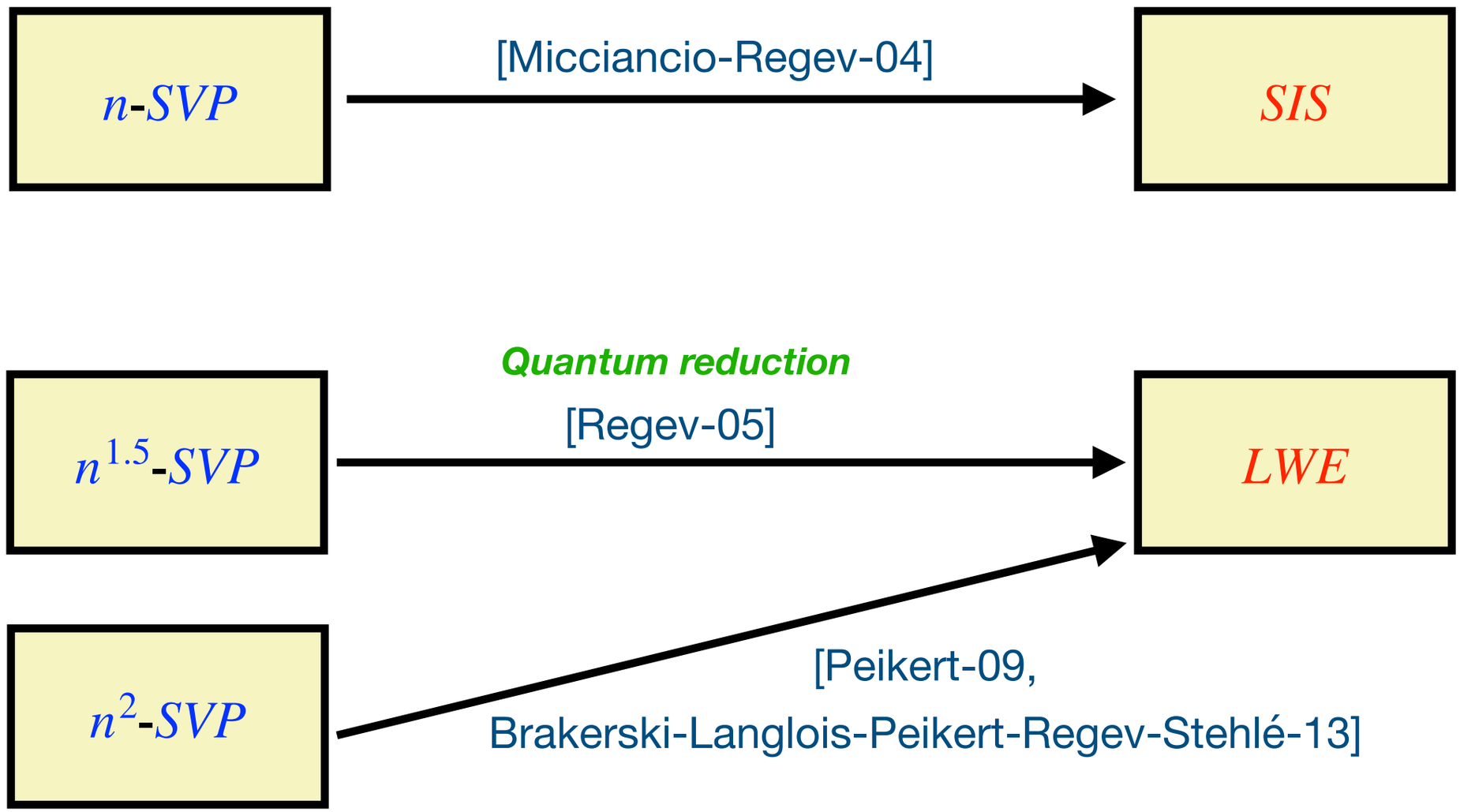
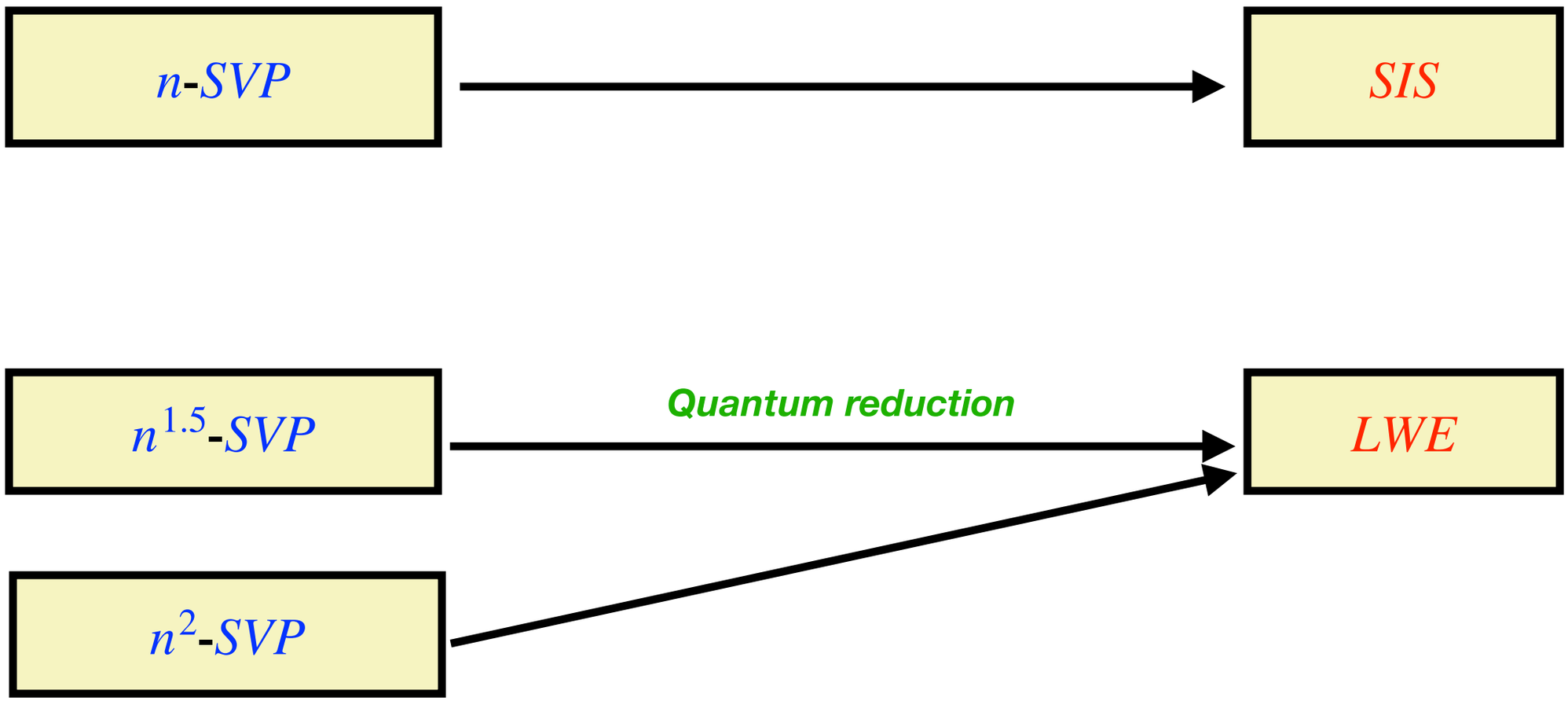


LWE and SIS

➡ We need exponential hardness of LWE and SIS for real-world security.



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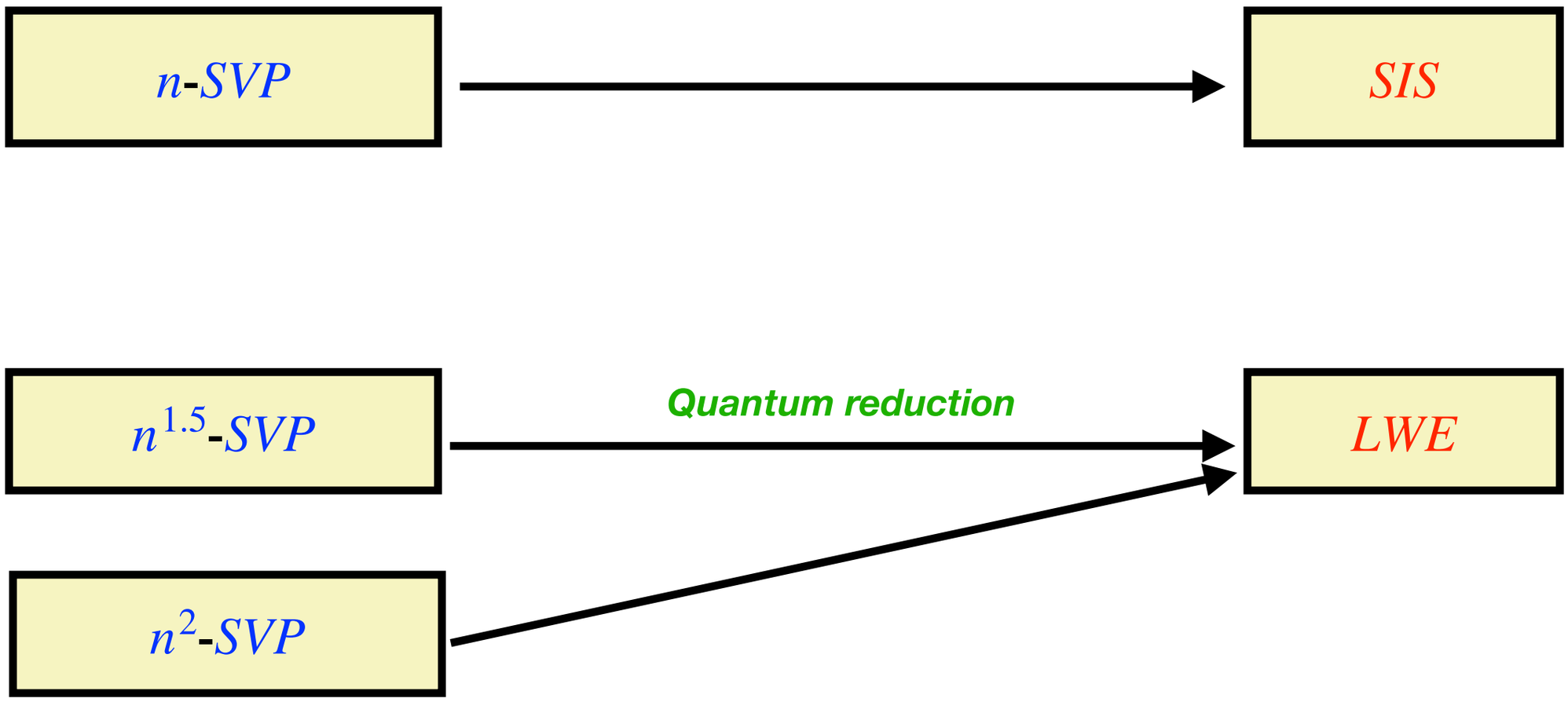

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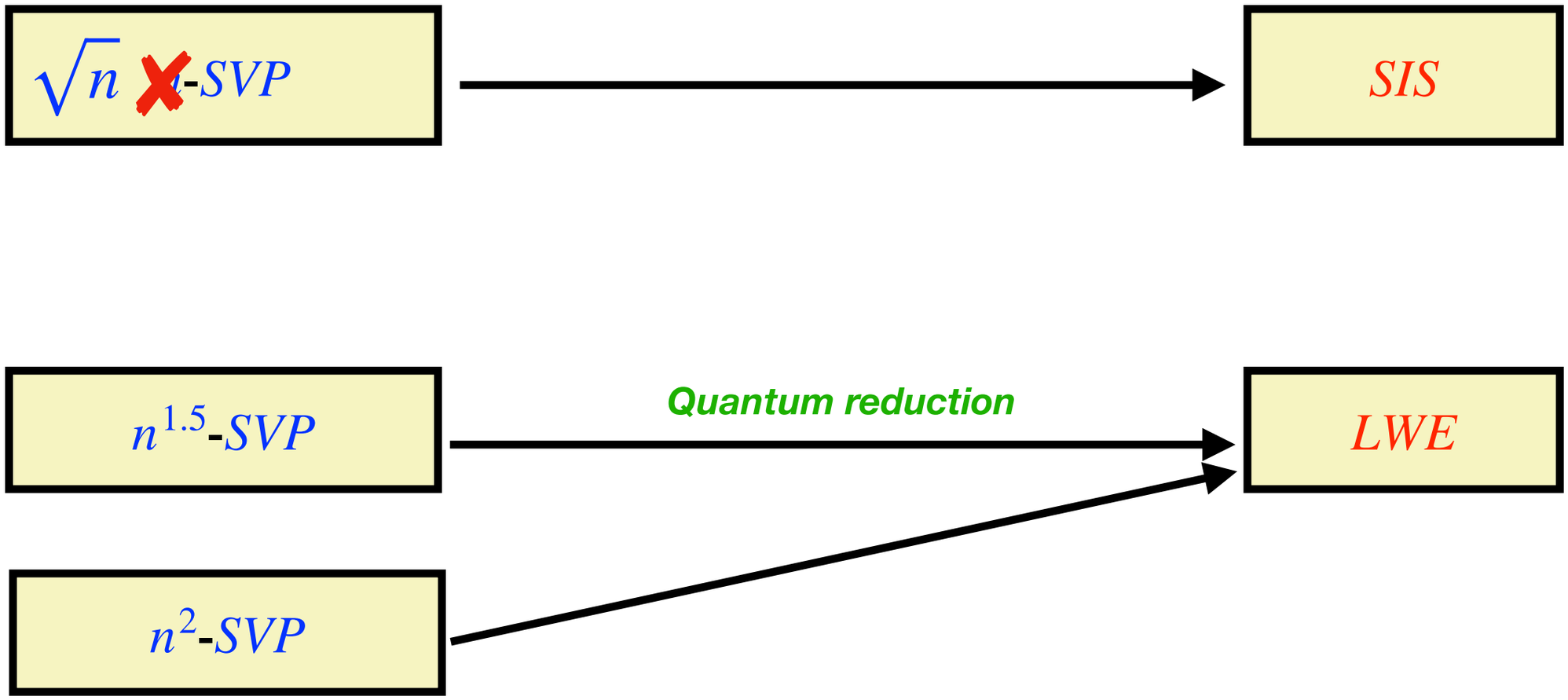

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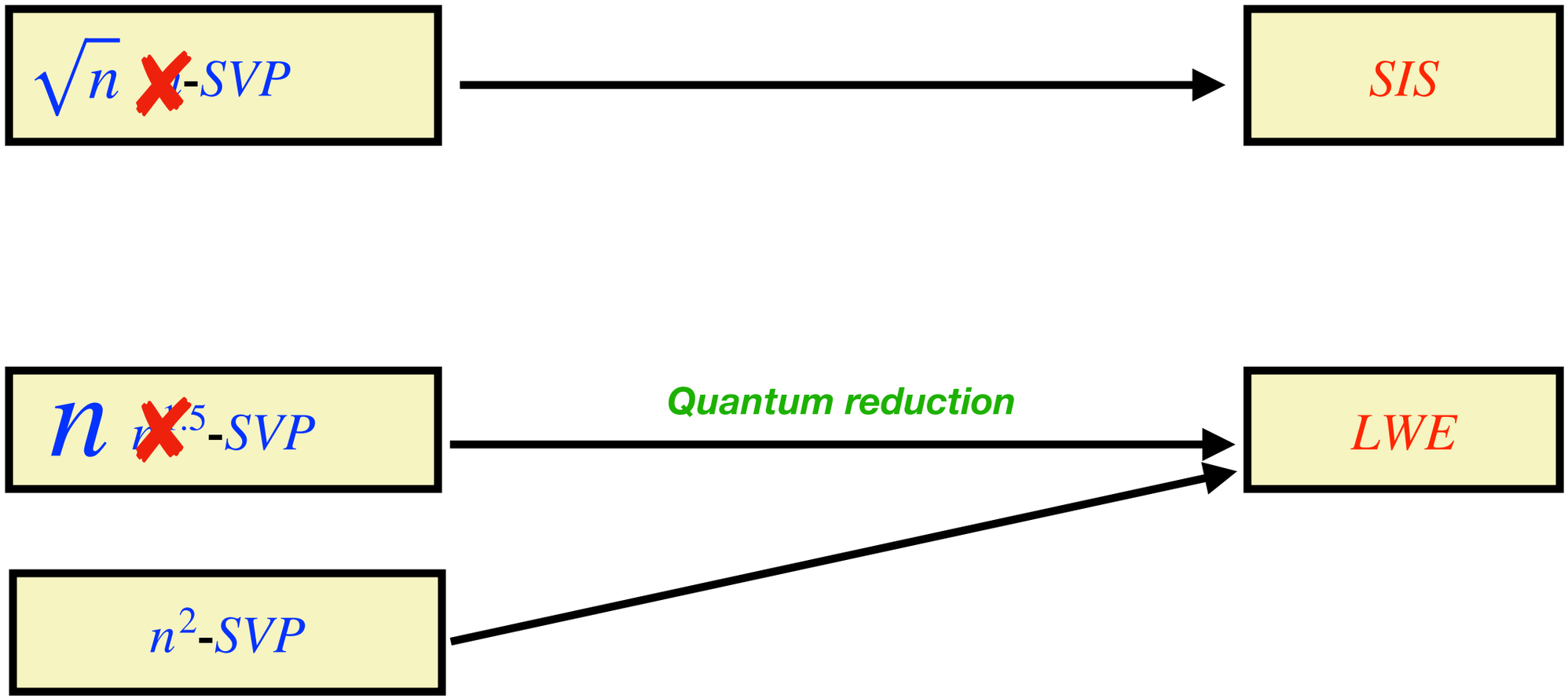

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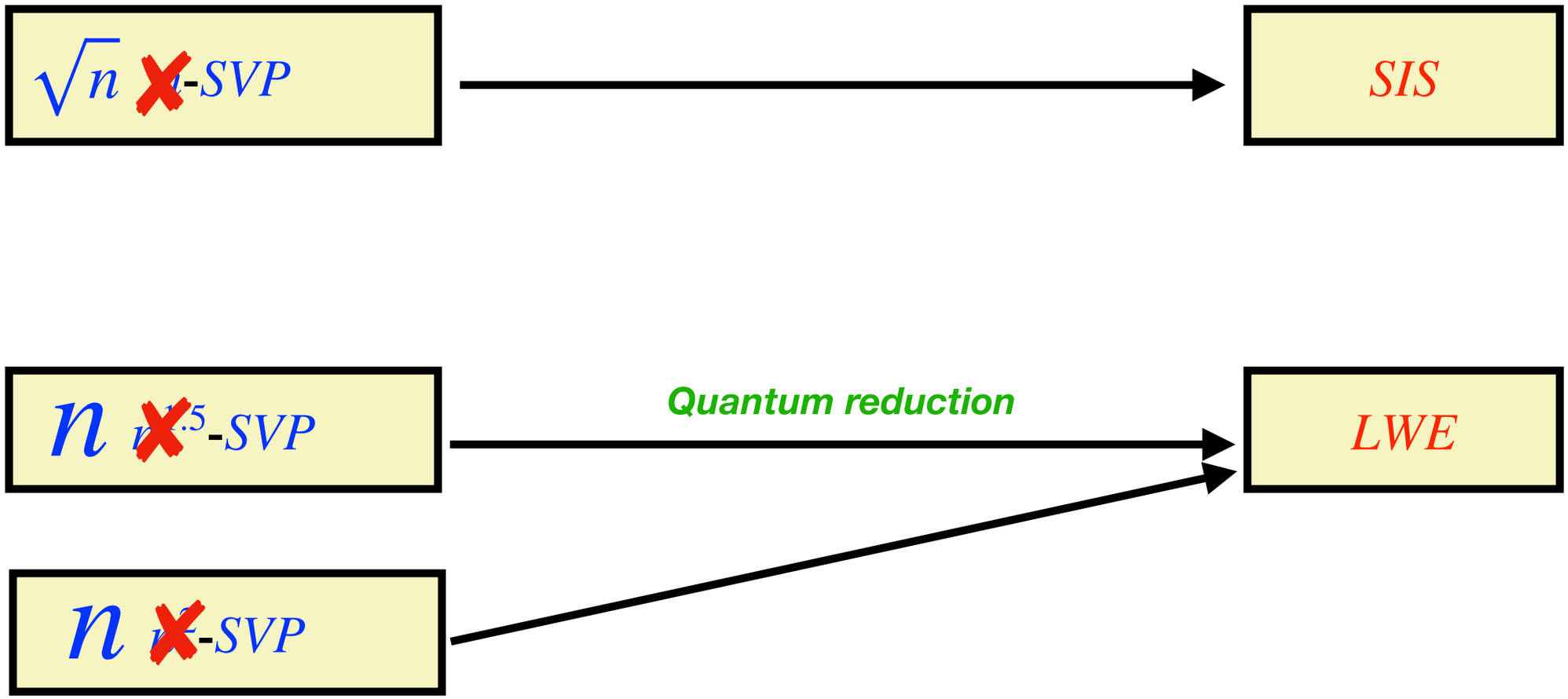

One more barrier!

➡ Can we show the exponential hardness of n^{ϵ}-SVP/CVP ?

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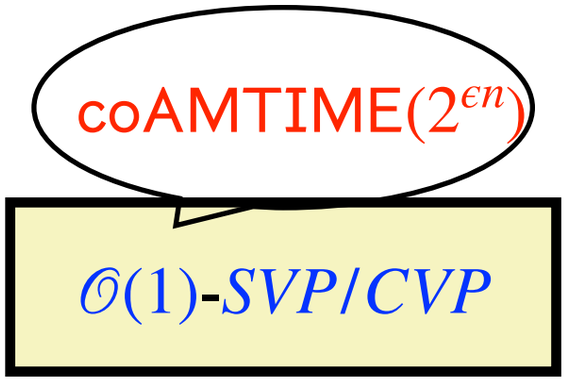

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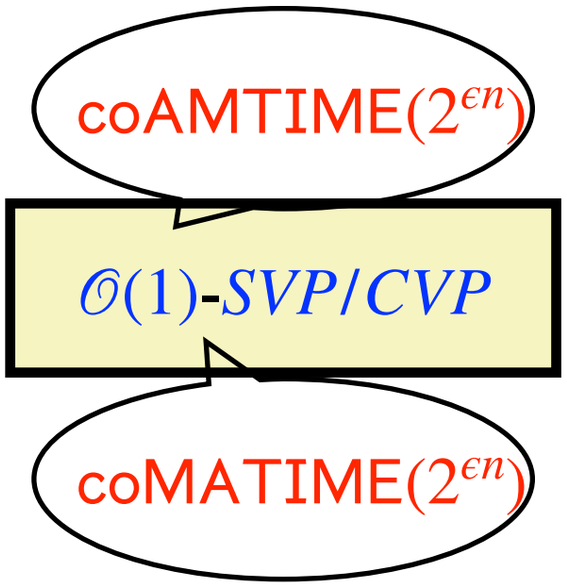

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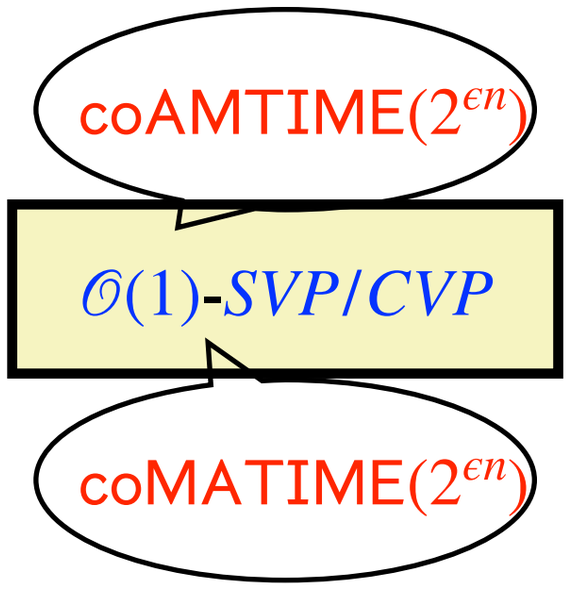

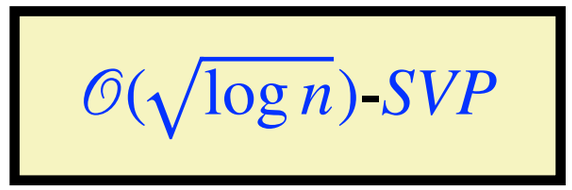

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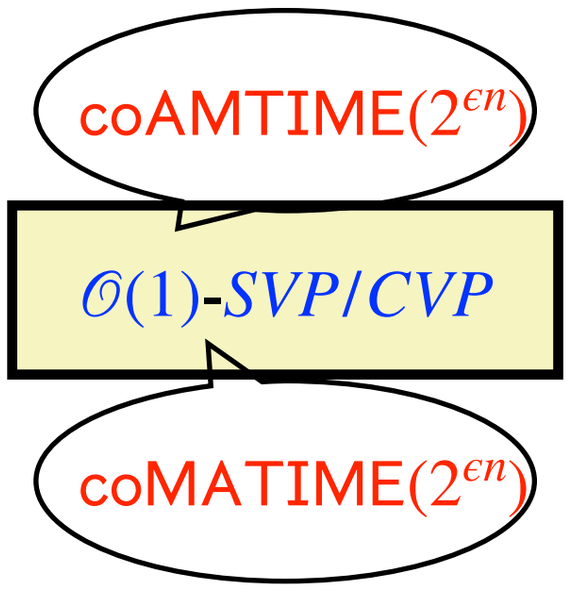

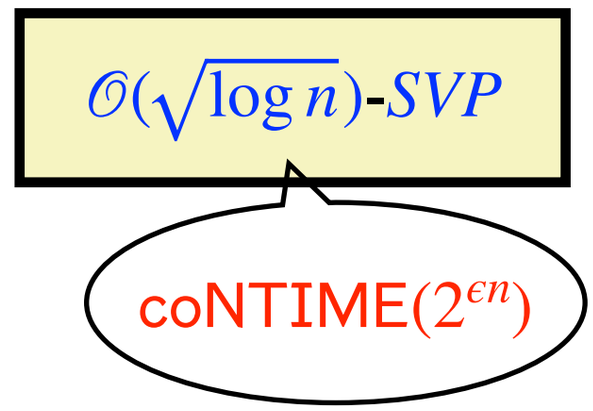

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It is conjectured that such protocols are not possible for k-SAT.

One more barrier!

Goldreich-Goldwasser-00

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Goldreich-Goldwasser-00

 coAMTIME(T) protocol for approx-CVP.

1. YES instance: If dist($\vec{t}, L$) $\leq d$

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(Computationally bounded)

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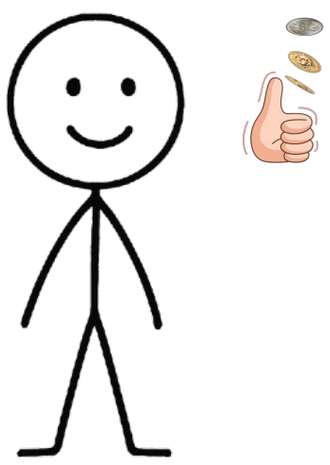

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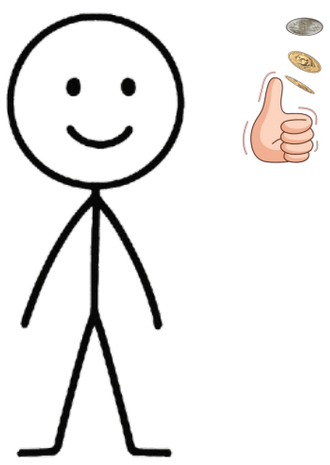

Arthur

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Query

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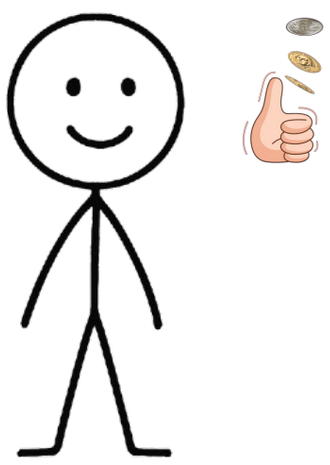

Query

Response

Arthur
(Computationally bounded)

Merlin
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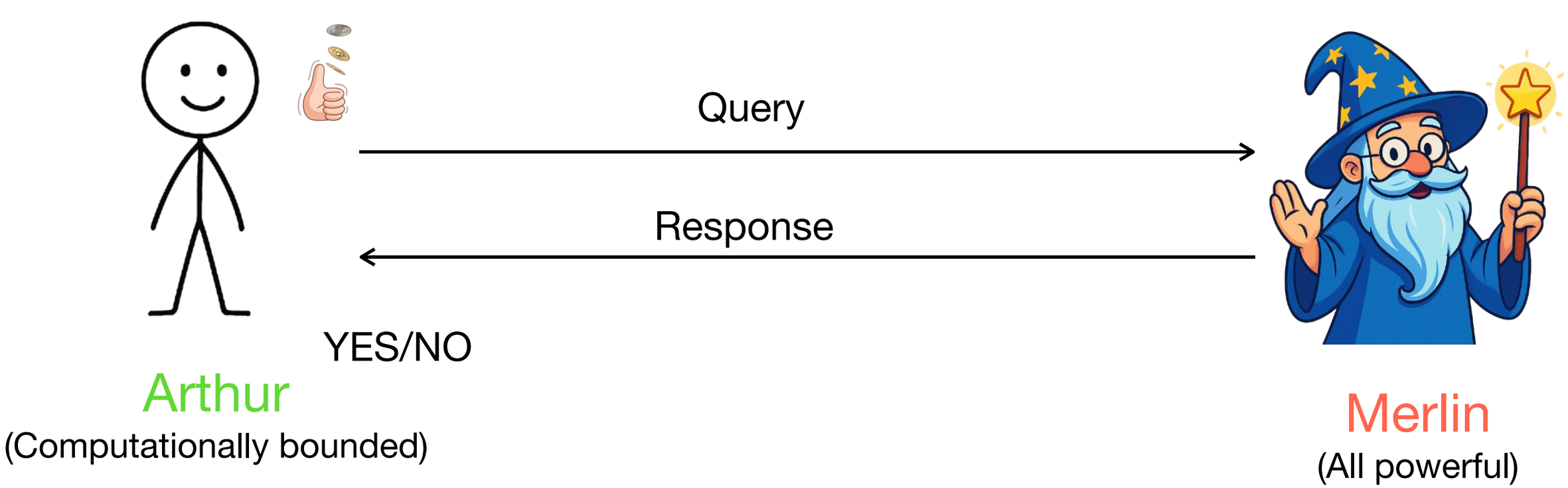

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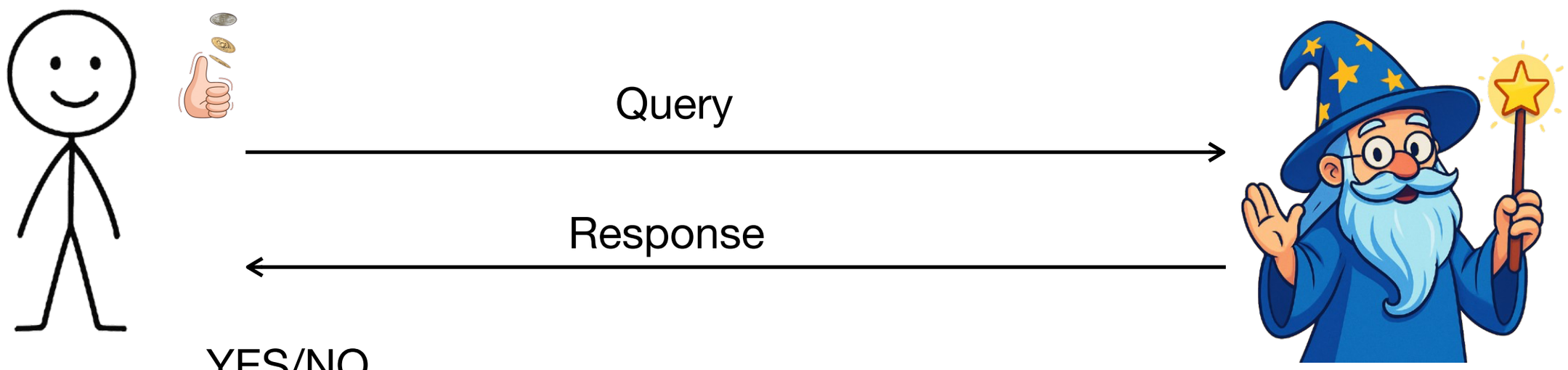
⇒ coAMTIME(T) protocol for approx-CVP.

1. YES instance: If dist($\vec{t}, L$) $\leq d$

2. NO instance: If dist($\vec{t}, L$) > 2 Merlin convinces Arthur that $\vec{t}$ is far from lattice.



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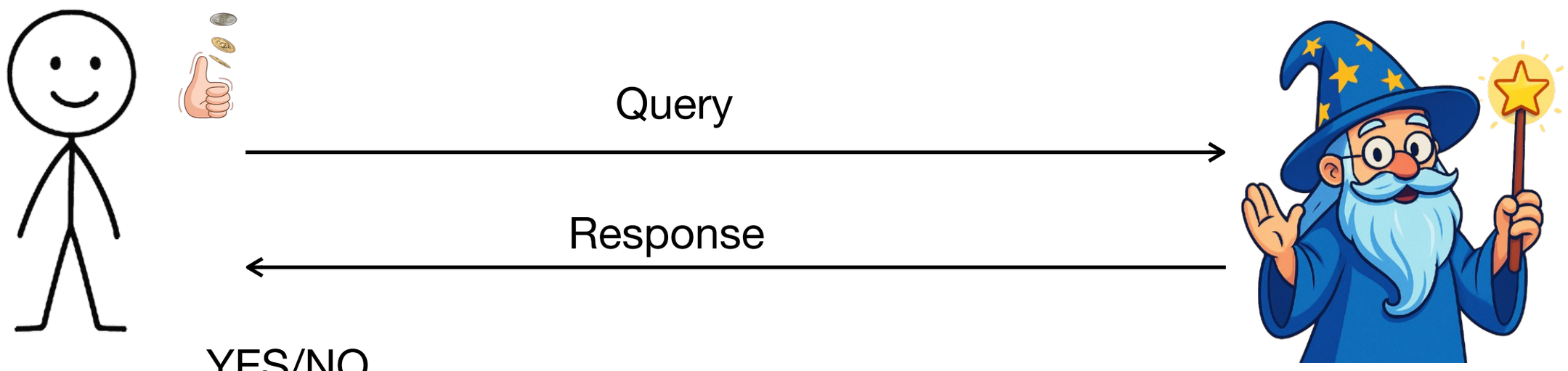
coAMTIME(T) protocol for approx-CVP.

1. YES instance: If $\text{dist}(\vec{t}, L) \leq d$ Merlin will not be able to convince Arthur that $\vec{t}$ is far from lattice.
2. NO instance: If $\text{dist}(\vec{t}, L) > 2$ Merlin convinces Arthur that $\vec{t}$ is far from lattice.



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$\mathscr{B} = \{\vec{x} \in \mathbb{R}^m \,|\, \|\vec{x}\| \leq 1\}$ (unit radius ball)

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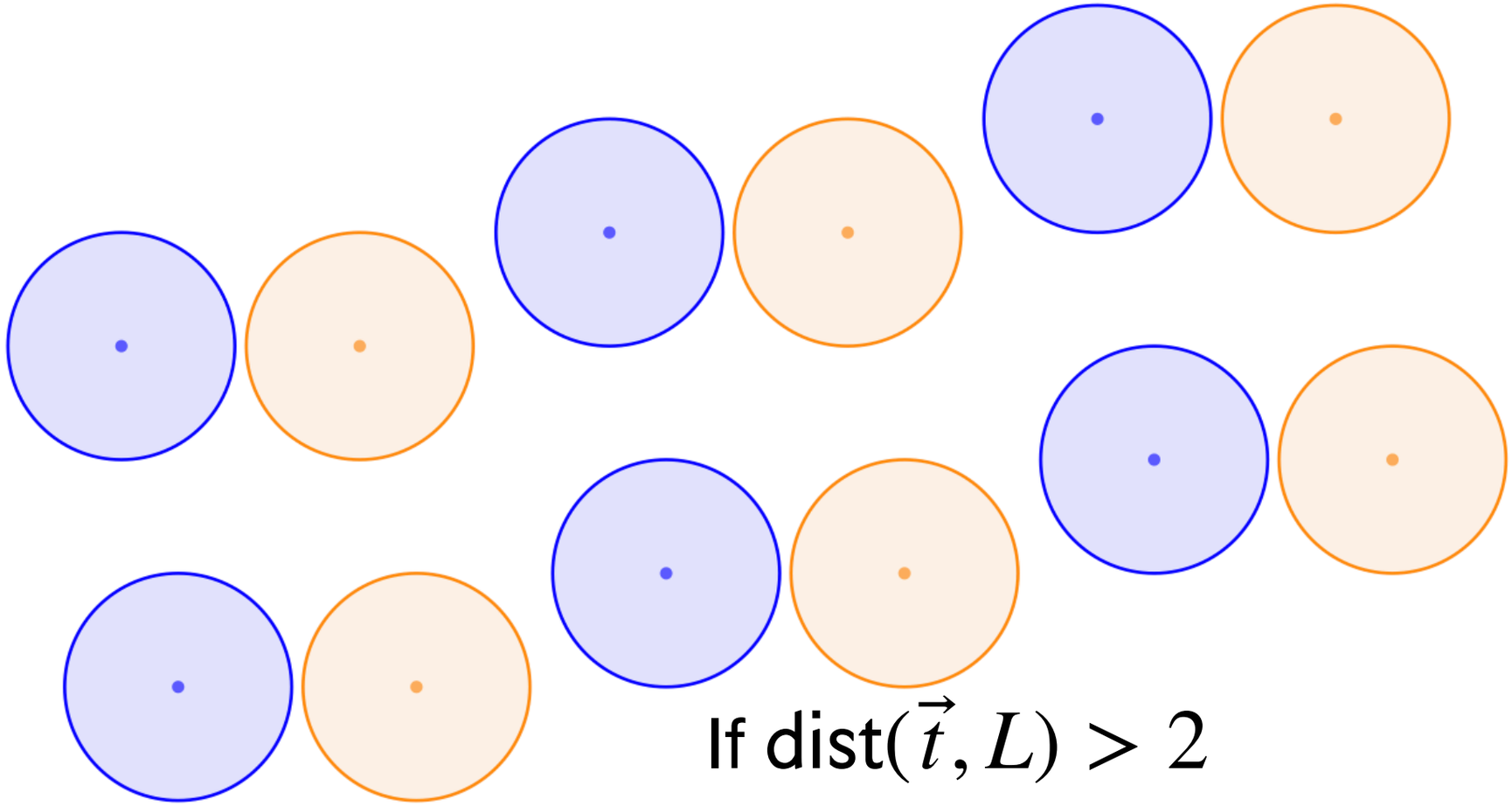

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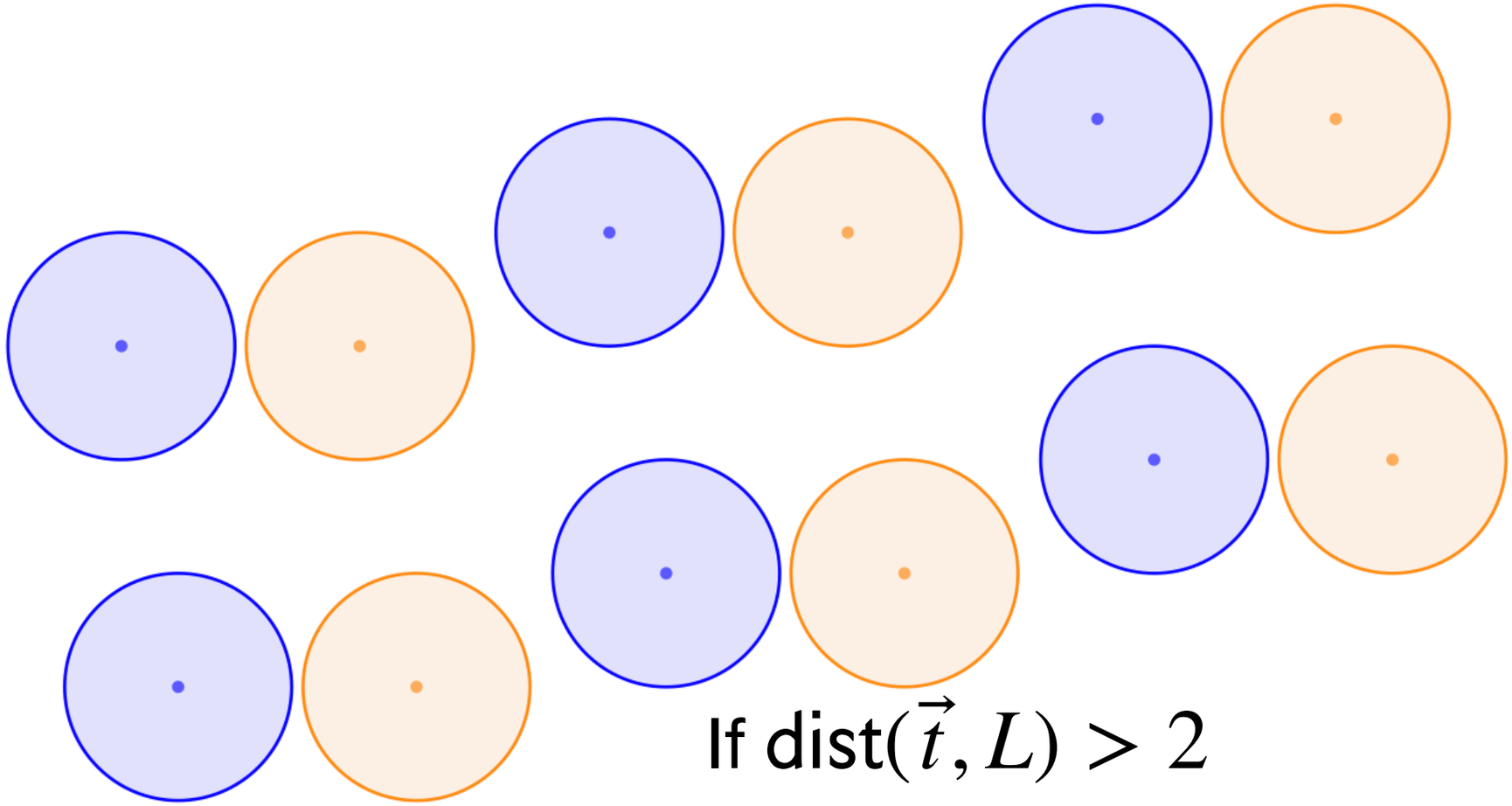

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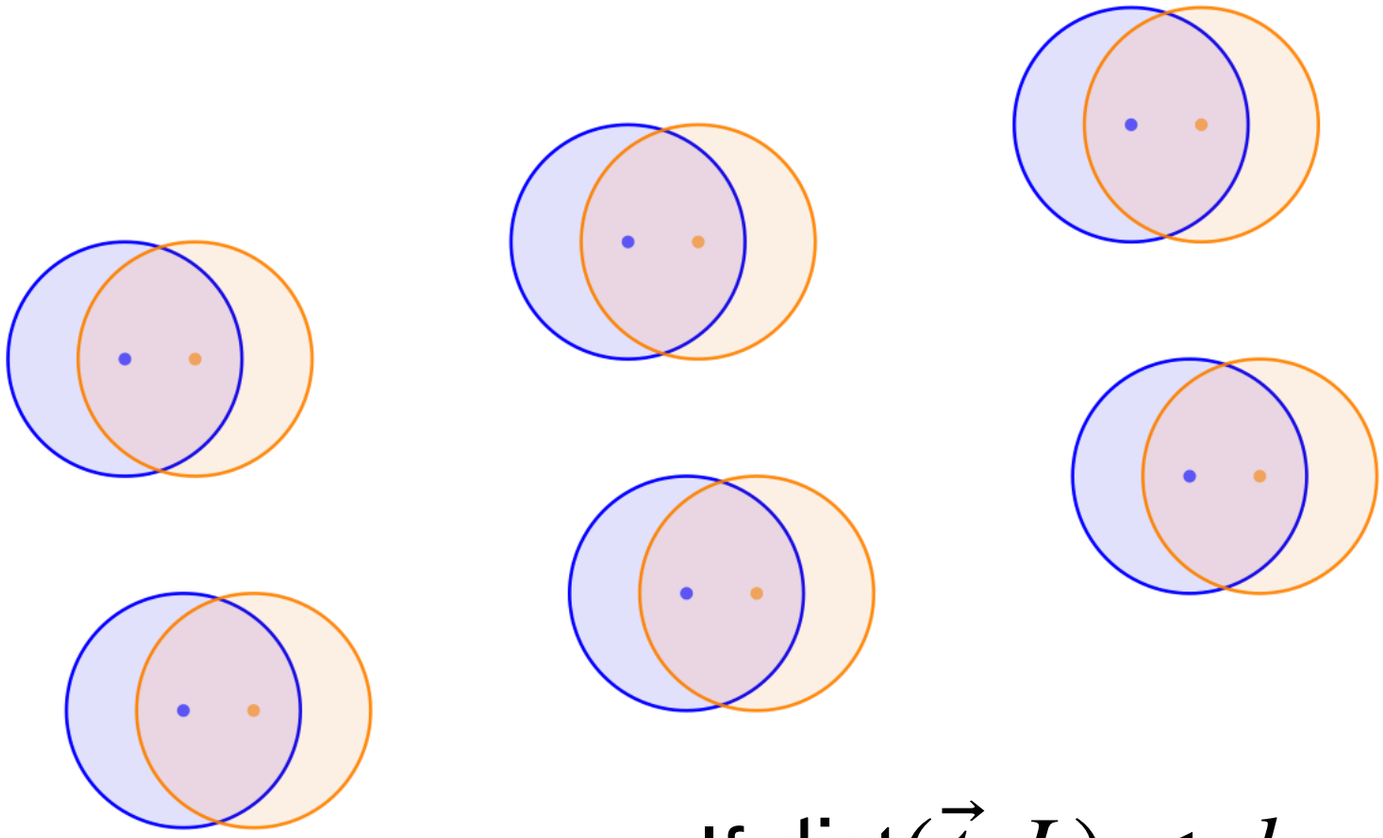

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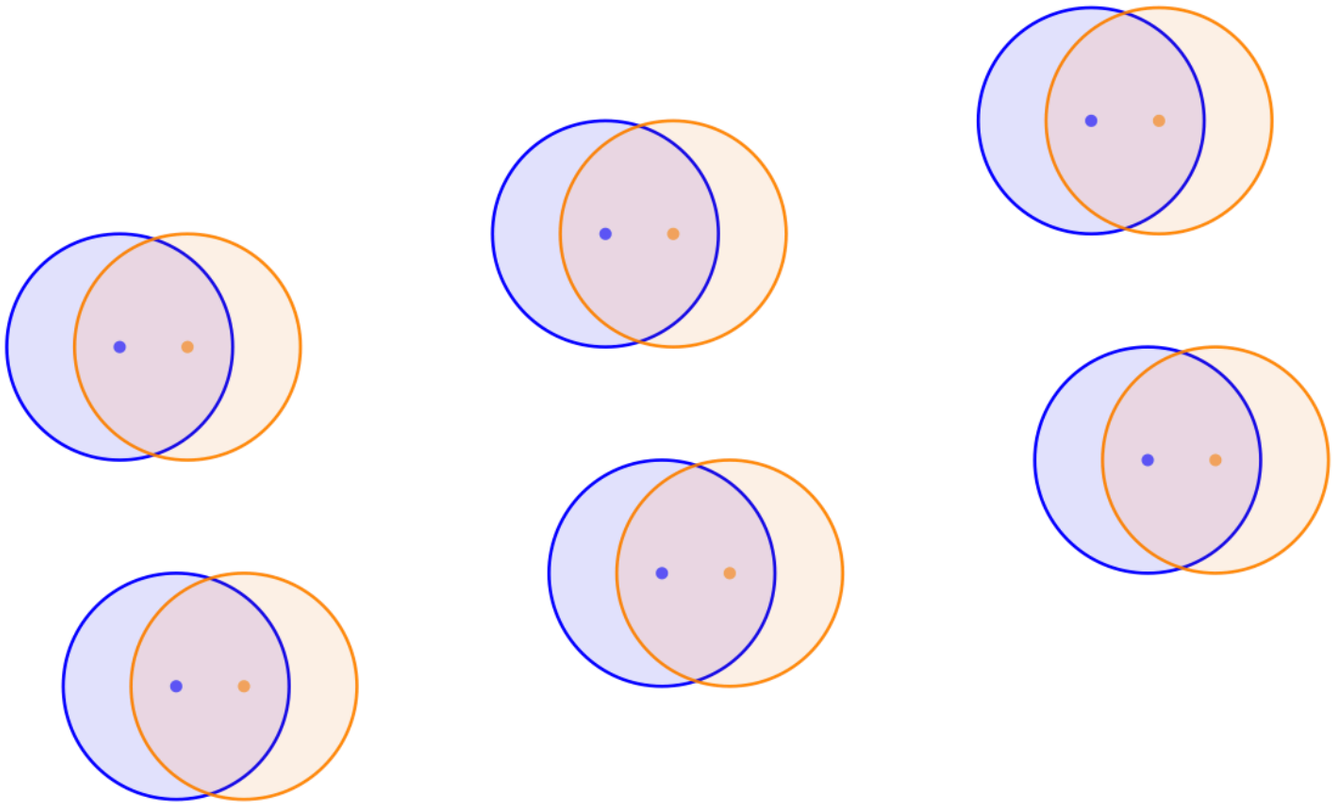

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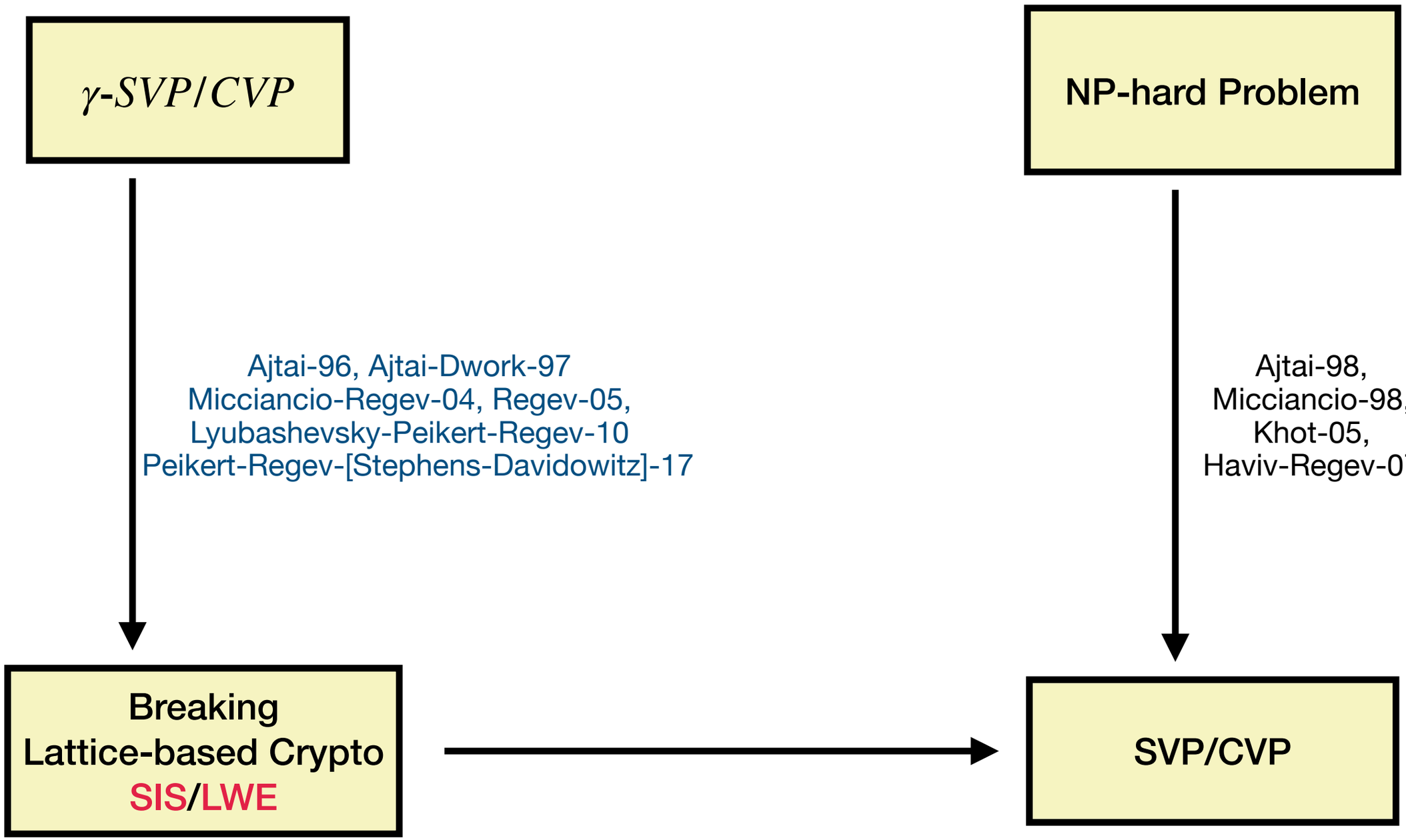
Dihedral Hidden Subgroup Problem

Conclusion


γ-SVP/CVP
Ajtai-96, Ajtai-Dwork-97
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Breaking
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Conclusion

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NP-hard Problem

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Thank you!